MATHEMATICS

Grade 8

Book 2

CAPS

Learner Book

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Table of contents

Term 3

Term 4

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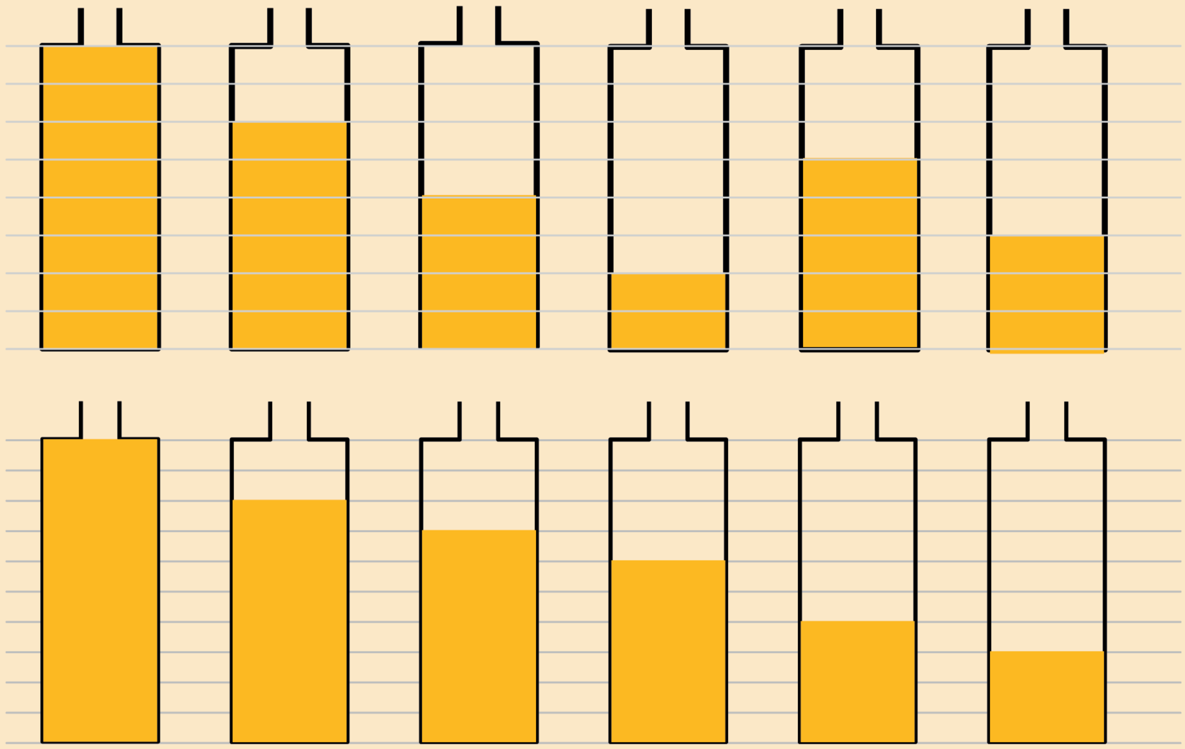
Chapter 1
Common fractions

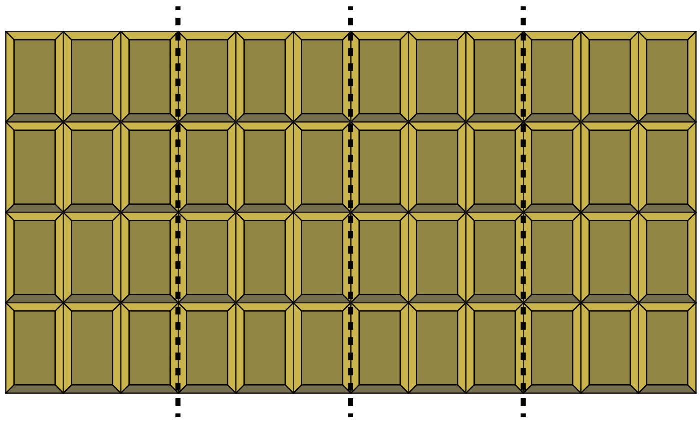
In this chapter you will learn more about fractions and what these numbers are used for. If we only use whole numbers we cannot always describe quantities precisely. Fractions were invented so that any quantity can be described accurately.



1 Common fractions

1.1 Equivalent fractions

SHARING CHOCOLATE IN DIFFERENT WAYS



1. (a) John eats three quarters of a chocolate slab like this one above. How many small pieces of chocolate is that?

   ....

   (b) How many small pieces are there in the whole slab of chocolate?

   ....

   (c) Ratti eats 6 eighths of a chocolate slab like the one above. Who eats more, Ratti or John, or do they eat the same amount of chocolate? Explain your answer.

   ....

   ....

2. A slab of chocolate like the above one has to be shared fairly between 16 people. That means each person should get **one sixteenth** of the slab. How many small pieces of chocolate should each person get? ....

3. What fraction of the whole slab is one of the small pieces? ....

4. (a) Is it true that each person in question 2 should get 1 sixteenth of the slab?

   ....

   (b) Is it true that each person in question 2 should get 3 forty-eighths of the slab?

   ....

   (c) Is 1 sixteenth of the slab of chocolate precisely the same amount of chocolate as 3 forty-eighths of the slab?

   ....

5. How many forty-eighths of a slab will each person get in each of the following cases, if the slab is equally shared among the number of people indicated?

(a) between 2 people .... (b) between 3 people ....

(c) between 4 people .... (d) between 6 people ....

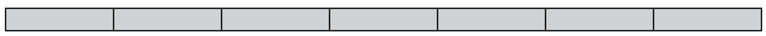
(e) between 8 people .... (f) between 12 people ....

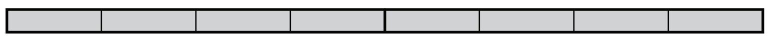
(g) between 16 people .... (h) between 24 people ....

6. In each case below, state what the smaller parts of the grey strip may be called.

(a)  ....

(b)  ....

(c)  ....

(d)  ....

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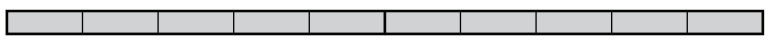
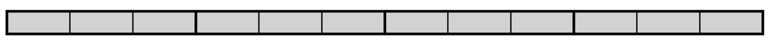
(f)  ....

(g)  ....

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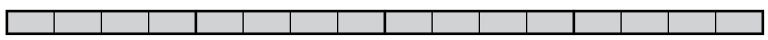
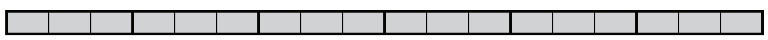
(i)  ....

(j)  ....

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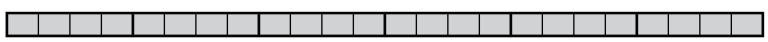
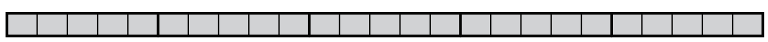
(l)  ....

(m)  ....

7. (a) A whole slab of chocolate is divided equally between a number of people, and each person gets 1 eighth of the slab. How many people are there?

....

(b) How many people are there if each person gets 1 twelfth of the slab?

....

(c) How many people are there if each person gets 1 sixteenth of the slab?

....

8. If each small piece is 1 forty-eighth of a slab of chocolate, how many pieces are there in each of the following?
   (a) 1 twelfth of a slab ....
   (b) 1 eighth of a slab ....
   (c) 1 third of a slab ....
   (d) 1 twenty-fourth of a slab ....
   (e) 1 sixth of a slab ....
   (f) 1 sixteenth of a slab ....

9. If each small piece is 1 forty-eighth of a slab of chocolate, how many pieces are there in each of the following?
   (a) 5 twelfths of a slab ....
   (b) 3 eighths of a slab ....
   (c) 2 thirds of a slab ....
   (d) 17 twenty-fourths of a slab ....
   (e) 5 sixths of a slab ....
   (f) 13 sixteenths of a slab ....

10. In each of the following say which fraction of the slab gives you more chocolate, or whether the two quantities are the same. How do you know this?
   (a) 5 sixths of a slab or 13 sixteenths of a slab

   ....

   ....

   (b) 5 twelfths of a slab or 3 eighths of a slab

   ....

   ....

   (c) 2 thirds of a slab or 17 twenty-fourths of a slab

   ....

   ....

11. (a) How many forty-eighths of a slab is 1 third of a slab and 1 eighth of a slab together?

   ....

   ....

(b) How much of a slab is 1 sixth of a slab and 3 eighths of a slab together?

....

....

(c) How much chocolate is 5 sixths of a slab and 7 eighths of a slab together?

....

....

12. (a) How many eighths of a slab is 18 forty-eighths of a slab? How did you work this out?

....

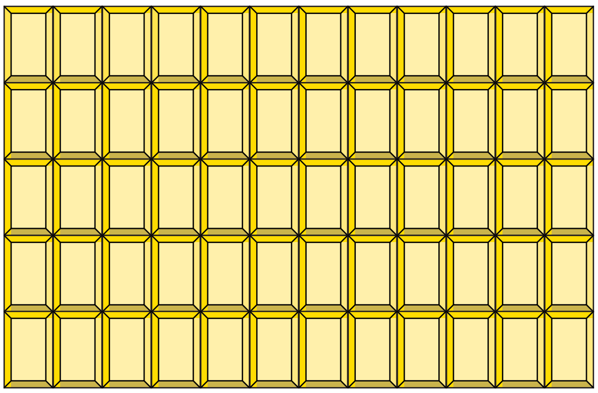
....

(b) How many sixths of a slab is 32 forty-eighths of a slab? How did you work this out?

....

....

Now here is a different slab of chocolate.



13. What fraction of the whole slab is each one of the small pieces? ....

14. How many sixtieths of the yellow 60-piece slab is each of the following?

(a) 1 fifth of the slab

....

(b) 1 twelfth of the slab

....

15. To answer question 14, you may just have counted the small pieces on the diagram. What calculations could you have done to find the answers for question 14?

....

16. How many sixtieths of the yellow 60-piece slab is each of the following?

(a) 1 twentieth of the slab

....

(b) 1 sixth of the slab

....

(c) 9 twentieths of the slab

....

17. In each case below, state which is more chocolate, or whether the two fractions of the slab are the same amount of chocolate. How do you know?

(a) 14 twentieths or 7 tenths

....

....

(b) 13 twentieths or 9 fifteenths

....

....

(c) 3 fifths or 7 twelfths

....

....

18. In each case below, work out how much of a slab is made up of the two parts together.

(a) 14 twentieths and 7 tenths. At the end, give your answer as a number of tenths.

....

....

(b) 13 twentieths and 9 fifteenths. Give your final answer as wholes and quarters.

....

....

(c) 3 fifths and 7 twelfths

....

....

USING FRACTION NOTATION

Instead of writing 5 forty-eighths, we may write $\frac{5}{48}$. This is called the **common fraction notation**.

The number 48 below the line is called the **denominator** and it shows that the whole was divided into 48 equal pieces, so each piece is 1 forty-eighth of the whole. The denominator shows the **unit** in which the number is expressed.

The number 5 above the line is called the **numerator** and it indicates the **number** of pieces.

A number that is made up of a whole number and a fraction, like 2 and 3 fifths, can be written as a **mixed number**: $2\frac{3}{5}$.

1. Write each of the following numbers in fraction notation.

   (a) 7 twentieths ....

   (b) 3 and 5 eighths ....

   (c) 2 and 7 ninths ....

   (d) 1 and 7 tenths ....

2. Write each of the following numbers in words.

   (a) $\frac{23}{100}$ ....

   (b) $3\frac{5}{30}$ ....

   (c) $2\frac{5}{18}$ ....

   (d) $\frac{17}{25}$ ....

3. (a) The strip below is divided into five equal parts.
   What part of the whole strip is each of the five parts? ....

   (b) If you divide each fifth into six smaller equal parts, how many smaller parts will there be altogether? ....

   (c) What fraction of the whole strip is each of these smaller parts? ....

4. (a) The strip below is divided into 10 equal parts.
   What part of the whole strip is each of the 10 parts? ....

   (b) If you divide each tenth into four smaller equal parts, how many smaller parts will there be altogether? ....

(c) What fraction of the whole strip is each of these smaller parts?

....

(d) If you divide each tenth into five smaller equal parts, how many smaller parts will there be altogether? ....

(e) What fraction of the whole strip is each of these smaller parts?

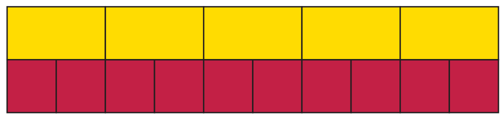
....

(f) If you divide each tenth into ten smaller equal parts, how many smaller parts will there be altogether? ....

(g) What fraction of the whole strip is each of these smaller parts?

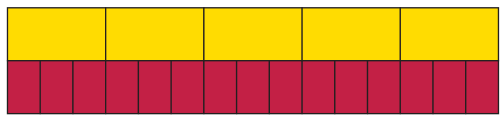
....

5. (a) How many tenths make up one fifth? You may use the diagram on the right to figure this out.



....

(b) How many fifteenths are there in one fifth?



....

(c) How many fifteenths are there in 3 fifths? ....

(d) How many twentieths are there in one fifth? If you need help with this, draw a diagram like those in questions 5(a) and (b) to help you. Your diagram need not be accurate.

....

(e) How many twentieths are there in one quarter? ....

(f) How many twentieths are there in 3 quarters? ....

(g) How many twentieths do you think will make up one tenth? If you need help, make marks on the diagram in question 5(a) to help you.

....

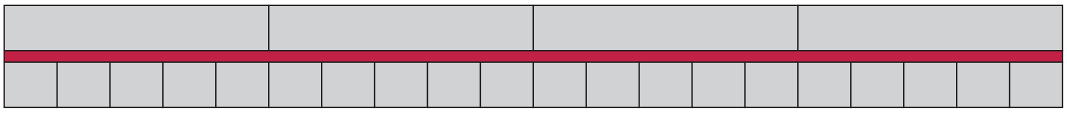
Your answers for question 5 can also be written in fraction notation. For example, your answer for 5(c) can be written as $\frac{3}{5} = \frac{9}{15}$.

6. Write each of your other answers for question 5 in fraction notation.

....

7. In this question write the fractions *in words*. Decide whether each statement is true or false and give reasons for your answers.

(a) "$\frac{15}{20}$ of the red strip below is longer than $\frac{3}{4}$ of the strip"



(b) "$\frac{9}{15}$ is a bigger number than $\frac{3}{5}$"

(c) "$\frac{2}{3}$ is a smaller number than $\frac{7}{12}$"

The same number can be expressed in different units. For example, the number $\frac{3}{4}$ can be expressed in eighths as $\frac{6}{8}$, in twentieths as $\frac{15}{20}$, in sixtieths as $\frac{45}{60}$ and in many other units. $\frac{3}{4}$, $\frac{6}{8}$, $\frac{15}{20}$ and $\frac{45}{60}$ are all different ways of expressing the same number. Hence they are called **equivalent fractions**.

Equivalent fractions let us write the same number in different ways.

$\frac{3}{4} = \frac{6}{8} = \frac{15}{20} = \frac{45}{60}$

8. Write your answers in words and in fraction notation, and explain your answers.

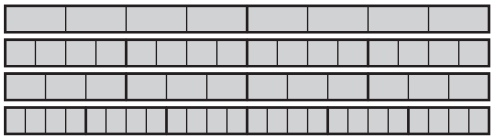
(a) Express $\frac{3}{8}$ in sixteenths and in fortieths.

(b) Express $\frac{3}{5}$ in tenths, twentieths, fortieths and hundredths.

(c) Express $\frac{7}{10}$ in fortieths, fiftieths and hundredths.

9. Consider the fraction 3 quarters. It can be written as $\frac{3}{4}$.

(a) Multiply both the numerator and the denominator by 2 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$? You may check on this diagram.



....

(b) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 3 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$? ....

(c) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 4 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$? ....

(d) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 6 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$? ....

$\frac{15}{20}$ is equivalent to $\frac{3}{4}$ because there are 5 twentieths in 1 quarter, and so there are 15 twentieths in 3 quarters. $\frac{9}{16}$ is not equivalent to $\frac{3}{4}$ because there are 4 sixteenths in 1 quarter, so 3 quarters is 12 sixteenths, not 9 sixteenths.

10. Decide whether the two given numbers are equal or not. Explain your answer. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that helps you.

(a) $\frac{5}{8}$ and $\frac{3}{5}$ (Hint: express both numbers in fortieths)

....

(b) $\frac{7}{10}$ and $\frac{5}{8}$

....

(c) $\frac{4}{5}$ and $\frac{7}{8}$

....

1.2 Adding and subtracting fractions

To add or subtract fractions, all the fractions must be expressed in the same unit.

1. Calculate each of the following. The work that you did in question 10 on the previous page may help you.
   (a) $\frac{5}{8} + \frac{3}{5} =$ ....
   (b) $\frac{7}{10} + \frac{5}{8} =$ ....
   (c) $\frac{7}{10} + \frac{3}{8} =$ ....
   (d) $\frac{5}{8} - \frac{3}{5} =$ ....
   (e) $\frac{7}{10} - \frac{3}{8} =$ ....
   (f) $6 \times \frac{5}{8}$ (which is $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$)

   ....
   (g) $8 \times \frac{7}{10}$ ....

To compare, add or subtract fractions, for example $\frac{5}{8}$ and $\frac{3}{5}$, find a fraction unit in which both fractions can be expressed so that you can compare them. We call this a **common denominator**. The "product" of the two denominators is helpful to find such a unit. In this case, $5 \times 8 = 40$. Since 1 eighth is 5 fortieths, $\frac{5}{8}$ is 25 fortieths or $\frac{25}{40}$. Since 1 fifth is 8 fortieths, $\frac{3}{5}$ is 24 fortieths or $\frac{24}{40}$. So, $\frac{5}{8}$ is bigger than $\frac{3}{5}$.

2. In each question explain why the two given numbers are equal or why they are not equal. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that will help you.
   (a) $\frac{5}{8}$ and $\frac{2}{3}$

   ....
   (b) $\frac{5}{6}$ and $\frac{7}{8}$

   ....
   (c) $\frac{3}{4}$ and $\frac{4}{5}$

   ....

(d) $\frac{5}{12}$ and $\frac{2}{3}$

....

(e) $\frac{7}{12}$ and $\frac{3}{8}$

....

(f) $\frac{9}{20}$ and $\frac{4}{15}$

....

(g) $\frac{3}{10}$ and $\frac{1}{4}$

....

(h) $\frac{7}{10}$ and $\frac{5}{8}$

....

(i) $\frac{9}{13}$ and $\frac{11}{17}$

....

3. Add the two fractions given in each part of question 2. Show how you work it out.

(a) $\frac{5}{8} + \frac{2}{3}$ ....

(b) $\frac{5}{6} + \frac{7}{8}$ ....

(c) $\frac{3}{4} + \frac{4}{5}$ ....

(d) $\frac{5}{12} + \frac{2}{3}$ ....

(e) $\frac{7}{12} + \frac{3}{8}$ ....

(f) $\frac{9}{20} + \frac{4}{15}$ ....

(g) $\frac{3}{10} + \frac{1}{4}$ ....

(h) $\frac{7}{10} + \frac{5}{8}$ ....

(i) $\frac{9}{13} + \frac{11}{17}$ ....

4. Now subtract the smaller number from the bigger number in each part of question 2.

(a) $\frac{2}{3} - \frac{5}{8}$ ....

(b) $\frac{7}{8} - \frac{5}{6}$ ....

(c) $\frac{4}{5} - \frac{3}{4}$ ....

(d) $\frac{2}{3} - \frac{5}{12}$ ....

(e) $\frac{7}{12} - \frac{3}{8}$ ....

(f) $\frac{9}{20} - \frac{4}{15}$ ....

(g) $\frac{3}{10} - \frac{1}{4}$ ....

(h) $\frac{7}{10} - \frac{5}{8}$ ....

(i) $\frac{9}{13} - \frac{11}{17}$ ....

5. Calculate each of the following.

(a) $3\frac{2}{3} - 1\frac{5}{6}$ ....

(b) $5\frac{6}{7} + \frac{3}{8}$ ....

(c) $12\frac{5}{8} + 7\frac{4}{9}$ ....

(d) $4\frac{5}{12} - 2\frac{3}{10}$ ....

(e) $1\frac{3}{10} - \frac{2}{3}$ ....

(f) $2\frac{7}{15} - 1\frac{3}{8}$ ....

(g) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$ ....

(h) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$ ....

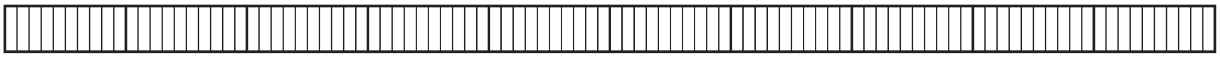
(i) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$ ....

(j) $2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12}$ ....

1.3 Tenths and hundredths and thousandths

A USEFUL FAMILY OF FRACTION UNITS

1. (a) Shade 3 tenths of the strip below.



(b) Into how many smaller parts is each tenth of the above strip divided? ....

(c) How many of these smaller parts are there in the whole strip? ....

(d) What is each of these smaller parts called? ....

(e) How many hundredths make up 2 fifths of the strip? ....

(f) How many hundredths make up 1 quarter of the strip? ....

(g) Shade 37 hundredths of the strip below.

2. Express each of the following numbers as a number of hundredths, and write your answers in fraction notation.

(a) 4 fifths ....
(b) 1 twentieth ....
(c) 7 twentieths ....
(d) 1 twenty-fifth ....
(e) 17 twenty-fifths ....
(f) 7 fiftieths ....

Because 1 twentieth is 5 hundredths, 7 twentieths is 35 hundredths.
This can also be expressed in fraction notation: $\frac{35}{100} = \frac{7}{20}$.

$\frac{7}{20}$ is called the **simplest form** of $\frac{35}{100}$ because $\frac{35}{100}$ cannot be expressed with a smaller numerator than 7.

3. Express each of the following fractions in its simplest form.

(a) $\frac{75}{100}$ ....
(b) $\frac{60}{100}$ ....
(c) $\frac{65}{100}$ ....
(d) $\frac{90}{100}$ ....

4. Calculate each of the following, and express your answer in its simplest form.

(a) $\frac{3}{25} + \frac{4}{20}$ ....
(b) $\frac{6}{25} + \frac{6}{20}$ ....
(c) $\frac{7}{100} + \frac{9}{200}$ ....

5. (a) How much is $\frac{1}{100}$ of R400? ....

(b) How much is $\frac{7}{100}$ of R250? ....

(c) How much is $\frac{25}{100}$ of R600? ....

(d) How much is $\frac{1}{4}$ of R600? ....

(e) How much is $\frac{40}{100}$ of R700? ....

(f) How much is $\frac{2}{5}$ of R700? ....

Instead of writing $\frac{40}{100}$ of R700, we may write $\frac{40}{100} \times$ R700.

6. Explain why your answers for questions 5(e) and 5(f) are the same.

....

Another word for *hundredth* is *per cent*.
Instead of saying
Miriam received ***32 hundredths*** *of the prize money*,
we can say
Miriam received ***32 per cent*** *of the prize money.*
The symbol for per cent is %.

7. How much is 80% of each of the following?

(a) R900

....

(b) R650

....

(c) R250

....

(d) R3 400

....

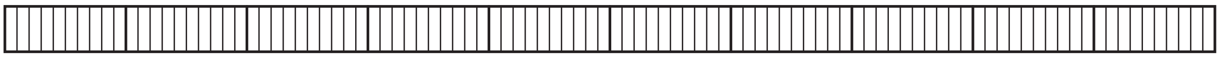
8. How much is 8% of each of the amounts in 7(a), (b), (c) and (d)?

....

9. How much is 15% of each of the amounts in 7(a), (b), (c) and (d)?

....

....



The above strip is divided into hundredths.

Imagine that each of the hundredths is divided into 10 equal parts (they will be almost impossible to see).

10. (a) How many of these very small parts will there be in the whole strip? ....

(b) What could each of these very small parts be called? ....

11. How much is each of the following?

(a) one tenth of R6 000 ....

(b) one hundredth of R6 000 ....

(c) one thousandth of R6 000 ....

(d) ten hundredths of R6 000 ....

(e) 100 thousandths of R6 000 ....

(f) 7 hundredths of R6 000 ....

(g) 70 thousandths of R6 000 ....

(h) one ten thousandth of R6 000 ....

12. Calculate.

(a) $\frac{3}{10} + \frac{5}{8}$ ....

(b) $3\frac{3}{10} + 2\frac{4}{5}$ ....

(c) $\frac{3}{10} + \frac{7}{100}$ ....

(d) $\frac{3}{10} + \frac{70}{100}$ ....

(e) $\frac{3}{10} + \frac{7}{1\,000}$ ....

(f) $\frac{3}{10} + \frac{70}{1\,000}$ ....

13. Calculate.

(a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ ....

(b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$ ....

(c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$ ....

(d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$ ....

Maths2_Gr8_LB_Book.indb 17 2014/09/04 10:38:27 AM

14. In each case investigate whether the statement is true or not, and give reasons for your decision.

(a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$ ....

....

....

(b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{46}{1\,000}$ ....

....

....

(c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$ ....

....

....

(d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$ ....

....

....

1.4 Fraction of a fraction

CALCULATE PARTS OF WHOLES AND PARTS OF PARTS

To calculate $\frac{7}{20}$ (7 twentieths) of R500 you can first calculate 1 twentieth, and then multiply by 7:

1 twentieth of R500 is R500 ÷ 20 = R25, so $\frac{7}{20}$ of R500 is 7 × R25 = R175.

This means that to calculate $\frac{7}{20}$ of R500 you work out (500 ÷ 20) × 7. You divide by the denominator and then multiply by the numerator.

$\frac{7}{20}$ of 500 is the same as $\frac{7}{20} \times 500$.

1. Calculate each of the following.
   (a) $\frac{9}{25}$ of R500 ....
   (b) $\frac{9}{20}$ of R500 ....
   (c) $\frac{9}{125}$ of R500 ....

2. A small choir of 8 members won the second prize in a competition and they received 2 fifths of the total prize money. They shared the money equally between themselves. The total prize money was R1 000. How much prize money did each member of the choir get?

   ....

   ....

3. (a) How much is $\frac{7}{8}$ of 400? ....
   (b) How much is $\frac{2}{5}$ of your answer for (a)? ....
   (c) How much is $\frac{7}{20}$ of 400? ....

4. Here is Nathi's answer to question 2:
   1 fifth of R1 000 is R200, so 2 fifths is R400. So the choir team received R400 in total. Each member received 1 eighth of the R400, which is R400 ÷ 8 = R50.
   (a) Compare your own answer to Nathi's answer. If they are different, work them out again and find out who is right.

   ....

   (b) Check whether you agree that $\frac{1}{20}$ of R1 000 is R50.

   ....

   (c) Try to explain why the answer for question 2 is the same as $\frac{1}{20}$ of R1 000.

   ....

   ....

   ....

   ....

   ....

5. Do the following for the numbers 80, 180, 260, 360 and 2 400. Do your work in the table given below.
   (a) How much is $\frac{3}{4}$ of each of the numbers?
   (b) How much is $\frac{2}{5}$ of each of your answers for (a)?
   (c) How much is $\frac{6}{20}$ of each of the numbers?

Number	80	180	260	360	2 400
$\frac{2}{5}$ of the number					
$\frac{3}{4}$ of the answer					
$\frac{6}{20}$ of the number					

6. Use your answers for question 5 to answer the following questions.
   (a) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R80? ....
   (b) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R180? ....
   (c) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R260? ....
   (d) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R360? ....
   (e) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R2 400? ....

7. To calculate $\frac{3}{4}$ *of* $\frac{2}{5}$ *of a number* you did this: *the number* ÷ *4* × *3* ÷ *5* × *2*.
   (a) Investigate whether *the number* × *3* × *2* ÷ *5* ÷ *4* will give the same results as *the number* ÷ *4* × *3* ÷ *5* × *2*, for the numbers in question 5 or any other numbers you may choose.

   ....

   (b) Investigate whether *the number* × *6* ÷ *20* will give the same results as *the number* × *3* × *2* ÷ *5* ÷ *4*.

   ....

   (c) Investigate whether *the number* × *3* ÷ *10* will give the same results as *the number* × *6* ÷ *20*.

   ....

Instead of $\frac{3}{4}$ of $\frac{2}{5}$ we may write $\frac{3}{4} \times \frac{2}{5}$.

$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5}$

To multiply by a mixed number like $2\frac{7}{8}$, it is good practice to express the whole number part in the same fraction units as the fraction part, for example:

2 wholes is 16 eighths, so $2\frac{7}{8}$ is $\frac{16}{8} + \frac{7}{8} = \frac{23}{8}$.

8. Calculate each of the following.

(a) $\frac{3}{10} \times \frac{12}{25}$ ....

(b) $\frac{5}{18} \times \frac{4}{35}$ ....

(c) $\left(\frac{1}{3}+\frac{1}{2}\right) \times \frac{6}{7}$

(d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$

....

(e) $2\frac{3}{5} \times \frac{5}{6}$

(f) $2\frac{3}{4} \times 3\frac{2}{5}$

....

(g) $2\frac{2}{3} \times 2\frac{2}{3}$

(h) $8\frac{2}{5} \times 3\frac{1}{3}$

....

(i) $\frac{6}{7} \times \left(\frac{1}{3}+\frac{1}{2}\right)$

(j) $\frac{6}{7} \times \frac{1}{3} + \frac{6}{7} \times \frac{1}{2}$

....

....

(k) $\frac{6}{7} \times \left(\frac{1}{2}-\frac{1}{3}\right)$

(l) $\frac{6}{7} \times \frac{1}{2} - \frac{6}{7} \times \frac{1}{3}$

....

(m) $\left(\frac{5}{6}+\frac{2}{3}\right) \times \frac{1}{5}$

(n) $\frac{5}{6} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{5}$

....

(o) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$

(p) $\frac{7}{8} \times \left(\frac{4}{7}+\frac{2}{5}\right)$

....

....

SQUARES AND CUBES AND ROOTS OF FRACTIONS

1. Calculate.

(a) $\frac{3}{10} \times \frac{3}{10}$

(b) $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

....

(c) $\left(\frac{3}{5}\right)^2$

(d) $\left(\frac{5}{9}\right)^2$

....

(e) $\left(\frac{3}{5}\right)^3$

(f) $\left(\frac{1}{4}\right)^2$

....

(g) $\left(\frac{1}{4}\right)^3$

(h) $\left(\frac{4}{7}\right)^2$

....

(i) $\left(\frac{5}{8}\right)^3$

(j) $\left(\frac{5}{8}\right)^2$

....

(k) $\left(\frac{5}{12}\right)^3$

(l) $\left(\frac{5}{12}\right)^2$

....

2. What number multiplied by itself will give $\frac{9}{16}$? ....

This number is called the square root of $\frac{9}{16}$. It can be written as $\sqrt{\frac{9}{16}}$.

3. Find each of the following. In some cases, your answers to question 1 will help you.

(a) $\sqrt{\frac{4}{9}}$ ....

(b) $\sqrt[3]{\frac{27}{64}}$ ....

(c) $\sqrt{\frac{25}{81}}$ ....

(d) $\sqrt[3]{\frac{125}{343}}$ ....

(e) $\sqrt{\frac{25}{36}}$ ....

(f) $\sqrt[3]{\frac{125}{216}}$ ....

(g) $\sqrt{\frac{9}{100}}$ ....

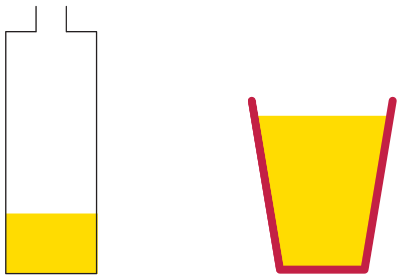
(h) $\sqrt[3]{\frac{27}{1\,000}}$ ....

1.5 Division by a fraction

SERVING JUICE

Jamie pours juice from bottles into glasses.

He uses three quarters of a bottle of juice to fill one glass.



1. How many bottles will Jamie need to fill 10 glasses?

   ....

2. How many bottles will Jamie need to fill 30 glasses?

   ....

3. How many bottles will Jamie need to fill 100 glasses?

   ....

4. How many bottles will Jamie need to fill 180 glasses?

   ....

5. How many bottles will Jamie need to fill 37 glasses?

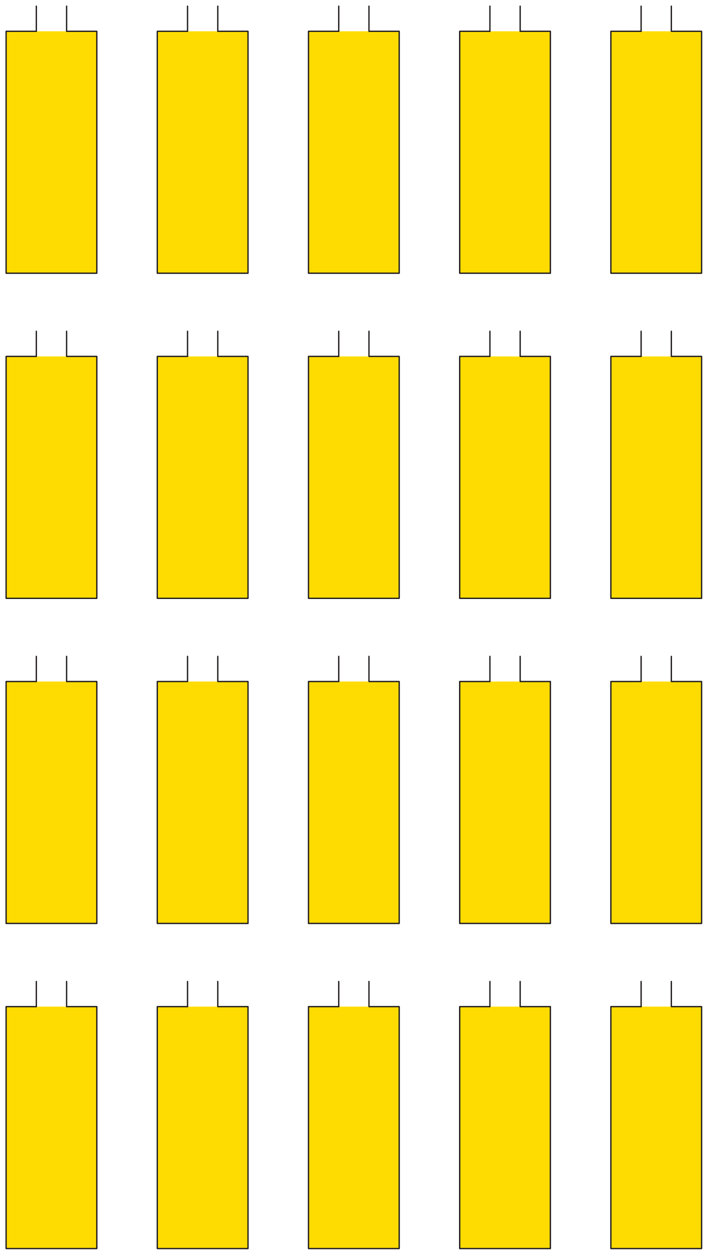
   ....

6. How many glasses can Jamie fill from 20 full bottles of juice?

....

....

....



7. How many glasses can Jamie fill from 36 full bottles of juice?

....

....

....

On another day Jamie uses different size glasses. He needs 5 eighths of a bottle of juice to fill one of these glasses.

8. How many bottles of juice does Jamie need to fill 50 of these glasses?

....

....

....

....

9. How many of these glasses can Jamie fill from 25 full bottles of juice?

....

....

....

....

Jamie changes glasses again. For the new glasses, he needs $\frac{7}{10}$ of a full bottle of juice to fill one glass.

10. How many bottles of juice does Jamie need to fill 44 of these glasses?

....

....

....

11. How many of these glasses can Jamie fill from 25 full bottles of juice?

....

....

....

....

12. How many glasses can Jamie fill from 36 full bottles of juice if he needs three-quarters of a bottle to fill one glass?

....

....

....

DOING THE JUICE CALCULATIONS MORE QUICKLY

1. Ria has R850 and chickens cost R67 each. What operation does she need to do to work out how many chickens she can buy?

    ....

2. Jamie has 16 bottles of juice and needs 3 quarters $\left(\frac{3}{4}\right)$ of a bottle to fill one glass.
    (a) How many quarters of a bottle of juice are there in 16 full bottles?

    ....

    (b) How many glasses can he fill with these quarters?

    ....

In question 2 you have worked out how many glasses, each taking $\frac{3}{4}$ of a bottle, can be filled from 16 bottles. You did this by first working out the total number of quarters in 16 bottles, and then dividing by 3 to find out how many glasses can be filled.
Do questions 3 and 4 in the same way.

3. Jamie has 20 bottles of juice and needs 5 eighths of a bottle to fill one glass. To work out how many glasses he can fill, he needs to work out 20 divided by $\frac{5}{8}$. Work in the same way you did for question 2 to find out.

    ....

    ....

4. Jamie has 25 bottles of juice and needs $\frac{3}{5}$ of a bottle to fill one glass. How many glasses can he fill?

    ....

    ....

    ....

    ....

In questions 2, 3 and 4 you have actually done the following calculations:

In question 2 you have calculated $16 \div \frac{3}{4}$, by doing $16 \times 4 \div 3$.

In question 3 you have calculated $20 \div \frac{5}{8}$, by doing $20 \times 8 \div 5$.

In question 4 you have calculated $25 \div \frac{3}{5}$, by doing $25 \times 5 \div 3$.

To divide by a fraction, you multiply by the denominator and divide by the numerator.

5. Calculate each of the following.

(a) $9 \div \frac{2}{3}$

(b) $12 \div \frac{3}{8}$

....

....

....

....

....

....

(c) $15 \div \frac{7}{10}$

(d) $2 \div \frac{3}{20}$

....

....

....

....

....

....

(e) $20 \div \frac{7}{12}$

(f) $120 \div 3\frac{3}{5}$

....

....

....

....

....

....

6. Calculate each of the following.

(a) $9 \times \frac{3}{2}$

(b) $12 \times \frac{8}{3}$

....

....

(c) $15 \times \frac{10}{7}$

(d) $2 \times \frac{20}{3}$

....

....

(e) $20 \times \frac{12}{7}$

(f) $120 \times \frac{5}{18}$

....

....

7. What do you notice about your answers for questions 5 and 6?

....

To divide by a fraction, we may turn the fraction around and multiply!

For example, $15 \div \frac{7}{10} = 15 \times \frac{10}{7}$.

$\frac{10}{7}$ is the **reciprocal** (also called the multiplicative inverse) of $\frac{7}{10}$.

Division is the inverse of multiplication.

The method of dividing by multiplying by the reciprocal also works when a fraction is divided by a fraction. For example $\frac{5}{18} \div \frac{7}{10}$ can be calculated by doing $\frac{5}{18} \times \frac{10}{7}$.

8. Calculate each of the following.

(a) $\frac{7}{10} \div \frac{3}{20}$

....

....

....

(b) $\frac{9}{10} \div \frac{3}{18}$

....

....

....

(c) $\frac{17}{20} \div \frac{2}{7}$

....

....

....

(d) $2\frac{7}{10} \div \frac{3}{5}$

....

....

....

(e) $4\frac{7}{8} \div \frac{2}{3}$

....

....

....

(f) $5\frac{7}{8} \div 2\frac{3}{5}$

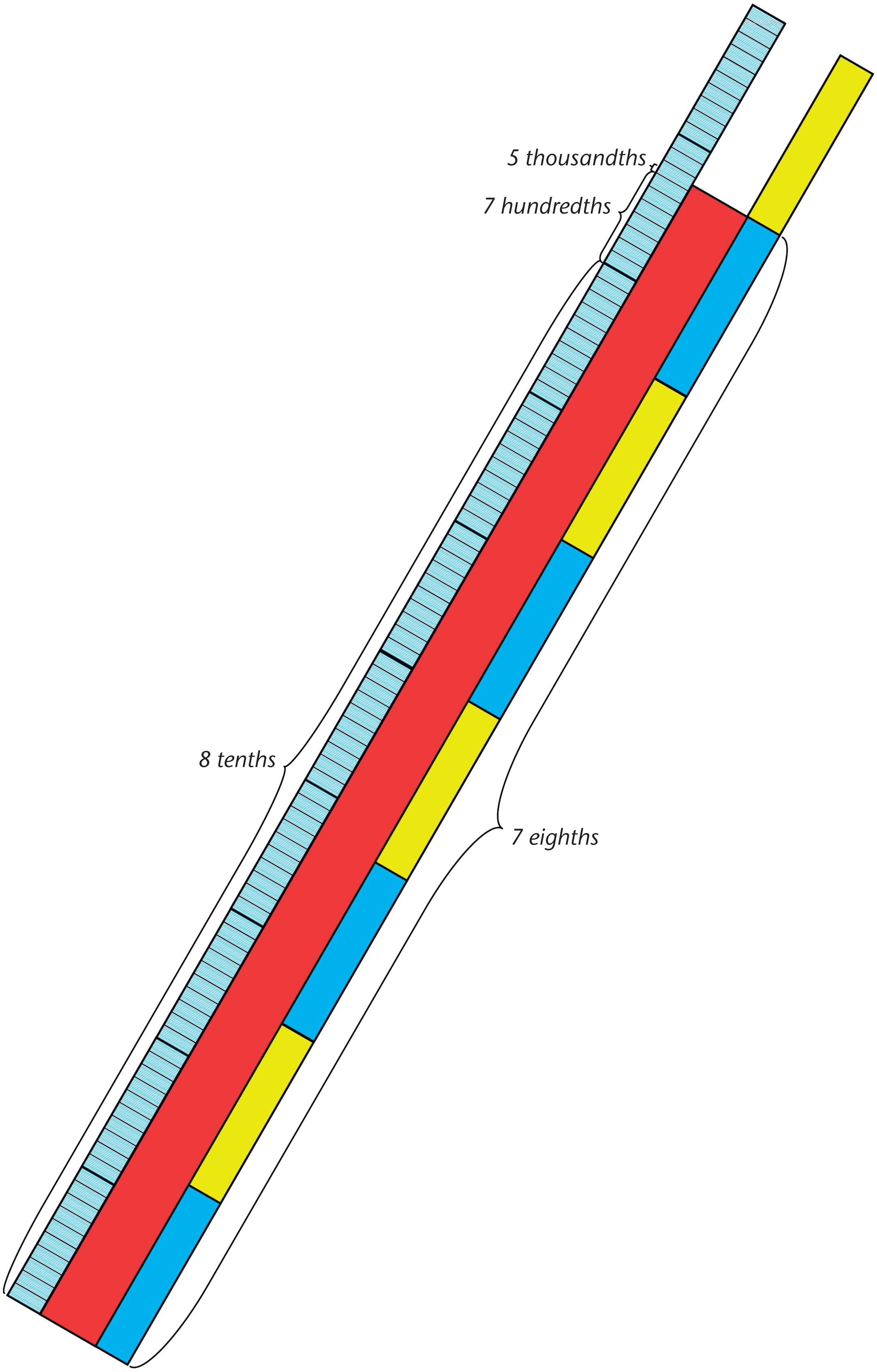
....

....

....

Chapter 2
Fractions in decimal notation

In this chapter you will do more work with fractions written in the decimal notation. When fractions are written in the decimal notation, calculations can be done in the same way as for whole numbers. It is important to always keep in mind that the common fraction form, the decimal form and the percentage form are just different ways to represent exactly the same numbers.

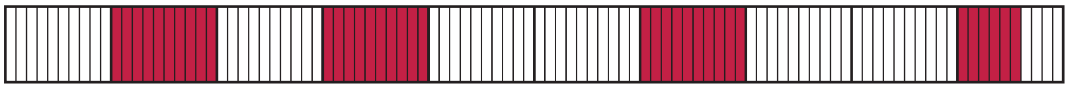

5 thousandths
7 hundredths
8 tenths
7 eighths

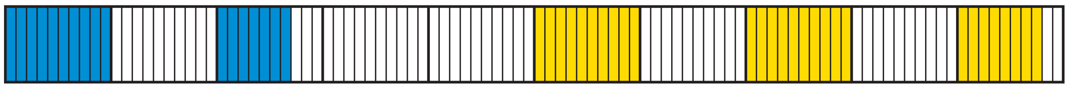
2 Fractions in decimal notation

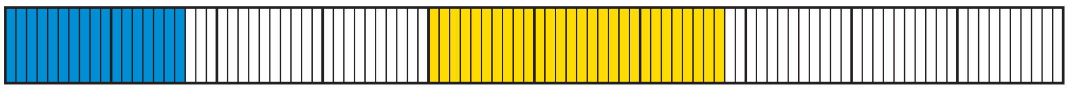
2.1 Equivalent forms

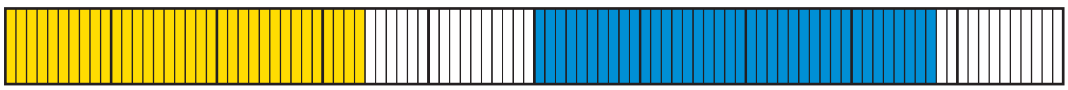
FRACTIONS IN DECIMAL NOTATION

1. What fraction of each rectangle is coloured in? Write your answers in the table.

(a) 

(b) 

(c) 

(d) 

Coloured in	Fraction notation	Decimal notation
(a) Red		
(b) Green		
Yellow		
(c) Green		
Yellow		
(d) Yellow		
Green		

2. Now find out what fraction in each rectangle in question 1 is not coloured in.

Not coloured in	Fraction notation	Decimal notation
(a)		
(b)		
(c)		
(d)		

Decimal fractions and common fractions are simply different ways of expressing the same number. We call them different **notations**.

To write a **common fraction as a decimal fraction**, we must first express the common fraction with a power of ten (10, 100, 1 000 etc.) as denominator.

For example: $\frac{9}{20} = \frac{9}{20} \times \frac{5}{5} = \frac{45}{100} = 0{,}45$

If you have a calculator, you can also divide the numerator by the denominator to get the decimal form of a fraction, for example: $\frac{9}{20} = 9 \div 20 = 0{,}45$

To write a **decimal fraction as a common fraction**, we must first express it as a common fraction with a power of ten as denominator and then simplify if necessary.

For example: $0{,}65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

3. Give the decimal form of each of the following numbers.

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{5}$ $\frac{7}{2}$ $\frac{65}{100}$

....

4. Write the following as decimal fractions.

(a) $2 \times 10 + 1 \times 1 + \frac{3}{10}$

....

....

(b) $3 \times 1 + 6 \times \frac{1}{100}$

....

....

(c) Three hundredths

....

(d) $7 \times \frac{1}{1\,000}$

....

5. Write each of the following numbers as fractions in their simplest form.

0,2 0,85 0,07 12,04 40,006

....

....

6. Write in the decimal notation.

(a) 5 + 12 tenths

....

(b) 2 + 3 tenths + 17 hundredths

....

(c) 13 hundredths + 15 thousandths

....

(d) 7 hundredths + 154 hundredths

....

HUNDREDTHS, PERCENTAGES AND DECIMALS

It is often difficult to compare fractions with different denominators. Fractions with the same denominator are easier to compare. For this and other reasons, fractions are often expressed as hundredths. A fraction expressed as hundredths is called a **percentage**.

Instead of 6 hundredths we can say 6 per cent or $\frac{6}{100}$ or 0,06.

6 per cent, $\frac{6}{100}$ and 0,06 are just three different ways of writing the same number.

The symbol % is used for per cent.

Instead of writing "17 per cent", we may write 17%.

1. Write each of the following in three ways: in decimal notation, in percentage notation and in common fraction notation. Leave your answers in hundredths.

(a) 80 hundredths

....

(b) 5 hundredths

....

(c) 60 hundredths

....

(d) 35 hundredths

....

2. Complete the following table.

Common fraction	Decimal fraction	Percentage
	0,3	
$\frac{1}{4}$		
		15%
$\frac{1}{8}$		
	0,55	
		1%

2.2 Ordering and comparing decimal fractions

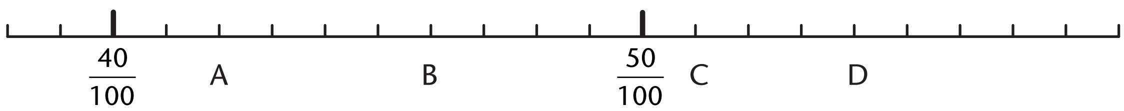
BIGGER, SMALLER OR THE SAME?

1. Write the values of the marked points (A to D) in as accurately as possible in *decimal notation*. Write the values *beneath* the letters A to D.

(a)

(b)

(c)



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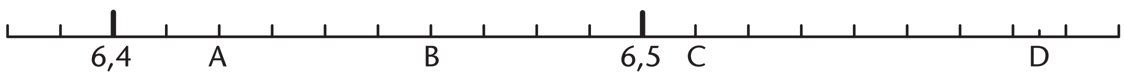

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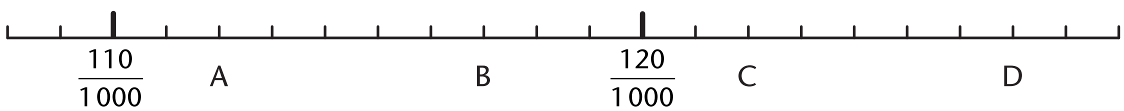

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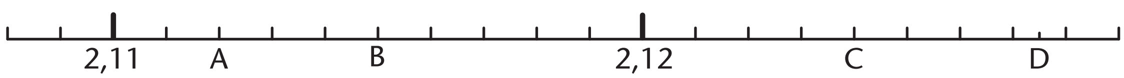
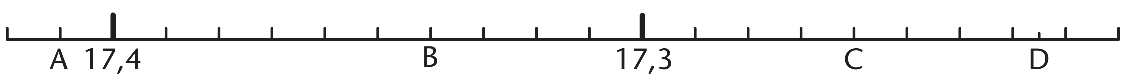
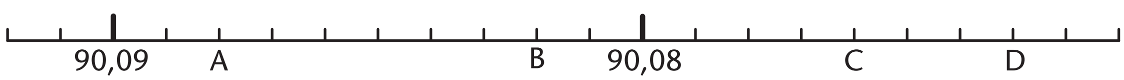


(g)

(h)



(i)



2. Order the following numbers from biggest to smallest. Explain your thinking.

5 267 1 263 1 300 12 689 635 1 267 125 126 12

....

....

....

3. Order the following numbers from biggest to smallest. Explain your method.

0,8	0,05	0,901	0,15	0,465	0,55	0,75	0,4	0,62
0,901	0,8	0,75	0,62	0,55	0,465	0,4	0,15	0,05

....

....

....

4. Write down *three* different numbers that are bigger than the first number and smaller than the second number.

(a) 5 and 5,1 .... (b) 5,1 and 5,11 .... (c) 5,11 and 5,12 ....

(d) 5,111 and 5,116 .... (e) 0 and 0,001 .... (f) $\frac{1}{2}$ and 1 ....

5. Underline the bigger of the two numbers.

(a) 2,399 and 2,6 (b) 5,604 and 5,64 (c) 0,11 and 0,087

(d) $\frac{3}{4}$ and 50% (e) $\frac{75}{100}$ and $\frac{50}{100}$ (f) 0,125 and 0,25

6. The table gives information about two world champion heavyweight boxers. If they fight against one another, who would you expect to have the advantage, and why?

	Wladimir Klitschko	**Alexander Povetkin**
Height (m)	1,98	1,88
Weight (kg)	112	103,3
Reach (m)	2,03	1,91

....

....

7. Fill in <, > or = .

(a) 3,09 ☐ 3,9 (b) 3,9 ☐ 3,90 (c) 2,31 ☐ 3,30

(d) 3,197 ☐ 3,2 (e) 4,876 ☐ 5,987 (f) 123,321 ☐ 123,3

8. How many numbers are there between 3,1 and 3,2?

....

Maths2_Gr8_LB_Book.indb 35 2014/09/04 10:39:03 AM

2.3 Rounding off decimal fractions

Decimal fractions can be rounded in the same way as whole numbers. They can be rounded to the nearest whole number or to one, two, three etc. figures after the comma.

If the last digit of the number is 5 or bigger it is rounded **up** to the next number. For example: 13,5 rounded to the nearest whole number is 14; 13,526 rounded to two figures after the comma is 13,53. If the last digit is 4 or less it is rounded **down** to the previous number. For example: 13,4 rounded to the nearest whole number is 13.

LET'S ROUND OFF

1. Round each of the following numbers off to the nearest whole number.
   29,34   3,65   14,452   3,299   39,1   564,85   1,768

   ....

2. Round each of the following numbers off to one decimal place.
   19,47   421,34   489,99   24,37   6,77

   ....

3. Round each of the following numbers off to two decimal places.
   8,345   6,632   5,555   34,239   21,899

   ....

4. Mr Peters buys a radio for R206,50. The shop allows him to pay it off over six months. How must he pay back the money?

   ....

   ....

5. (a) Mrs Smith buys a carton of 10 kg grapes at the market for R24,77. She must divide it between herself and two friends. How much does each woman get?

   ....

   (b) How much must each person pay Mrs Smith for the grapes?

   ....

   ....

6. Estimate the answers for each of the following by rounding off the numbers.
   (a) 1,43 × 1,62 .... (b) 3,89 × 4,21 ....

2.4 Calculations with decimal fractions

To **add** and **subtract** decimal fractions

- tenths may be added to tenths
- tenths may be subtracted from tenths
- hundredths may be added to hundredths
- hundredths may be subtracted from hundredths etc.

LET'S DO CALCULATIONS!

1. Four consecutive stages in a cycling race are 21,4 km; 14,7 km; 31 km and 18,6 km long. How long is the whole race?

   Answer: ....

2. Calculate.

   (a) 16,52 + 2,35
   (b) 16,52 + 9,38
   (c) 16,52 + 9,78
   (d) 30,08 + 2,9
   (e) 0,042 + 0,103
   (f) 9,99 + 0,99

3. Calculate.

   (a) 45,67 – 23,25
   (b) 45,67 – 23,80
   (c) 187,6 – 98,45
   (d) 1,009 – 0,998
   (e) 0,9 – 0,045
   (f) 65,7 – 37,6

Maths2_Gr8_LB_Book.indb 37 2014/09/04 10:39:03 AM

4. The following set of measurements (in cm) was recorded during an experiment:
56,8; 55,4; 78,9; 57,8; 34,2; 67,6; 45,5; 34,5; 64,5; 88

(a) Find the sum of the measurements and round it off to the nearest whole number.

....
....
....
....
....

(b) First round off each measurement to the nearest whole number and then find the sum.

....
....
....
....
....

(c) Which of your answers in 4(a) and (b) is closest to the actual sum? Explain why.

....
....

5. By how much is 0,7 greater than 0,07?

....
....
....

6. The difference between two numbers is 0,75.
The bigger number is 18,4.
What is the other number?

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To **multiply** fractions written as decimals, convert the fractions to whole numbers by multiplying by powers of 10 (e.g. 0,3 × 10 = 3), do your calculations with the whole numbers, and then convert back to decimals again.

For example: 13,1 × 1,01

13,1 **× 10** × 1,01 **× 100** = 131 × 101 = 13 231; 13 231 **÷ 10 ÷ 100** = 13,231

When you do **division** you can first multiply the number and the divisor by the same number to make the working easier.

For example: 21,7 ÷ 0,7 = (21,7 **× 10**) ÷ (0,7 **× 10**) = 217 ÷ 7 = 31

7. Calculate each of the following. You may use fraction notation if you wish.

(a) $0,12 \times 0,3$ (b) $0,12 \times 0,03$ (c) $1,2 \times 0,3$

(d) $350 \times 0,043$ (e) $0,035 \times 0,043$ (f) $0,13 \times 0,16$

(g) $1,3 \times 1,6$ (h) $0,13 \times 1,6$

8. $30,5 \times 1,3 = 39,65$. Use this answer to work out each of the following.

(a) $3,05 \times 1,3$ (b) $305 \times 1,3$ (c) $0,305 \times 0,13$

(d) 305×13 (e) $39,65 \div 30,5$ (f) $39,65 \div 0,305$

(g) $39,65 \div 0,13$ (h) $3,965 \div 130$

9. $3,5 \times 4,3 = 15,05$. Use this answer to work out each of the following.

(a) $3,5 \times 43$ (b) $0,35 \times 43$ (c) $3,5 \times 0,043$

(d) $0,35 \times 0,43$ (e) $15,05 \div 0,35$ (f) $15,05 \div 0,043$

10. Calculate each of the following. You may convert to whole numbers to make it easier.

(a) $62,5 \div 2,5$ (b) $6,25 \div 2,5$

(c) $6,25 \div 0,25$ (d) $0,625 \div 2,5$

2.5 Solving problems

1. (a) Divide R44,45 between seven people so that each one receives the same amount.

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   (b) John saves R15,25 every week. He now has R106,75 saved up. For how many weeks has he been saving?

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2. (a) Calculate $14,5 \div 6$, correct to two decimal places.

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   (b) Calculate $7,41 \div 5$, correct to one decimal place.

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3. Determine the value of x. (Give answers rounded to 2 decimal places.)

   (a) $7,1 \div x = 4,2$

   ....

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   (b) $x \div 0,7 = 6,2$

   ....

   ....

   (c) $12 \div x = 6,4$

   ....

   ....

   (d) $x \div 3,5 = 7$

   ....

   ....

   (e) $2,3 \times x = 6$

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   ....

   (f) $0,023 \times x = 8$

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4. (a) 1 ℓ of water weighs almost 0,995 kg. What will 50 ℓ of water weigh? What will 0,5 ℓ of water weigh?

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   (b) Mincemeat costs R36,65 per kilogram. What will 3,125 kg mincemeat cost? What will 0,782 kg cost?

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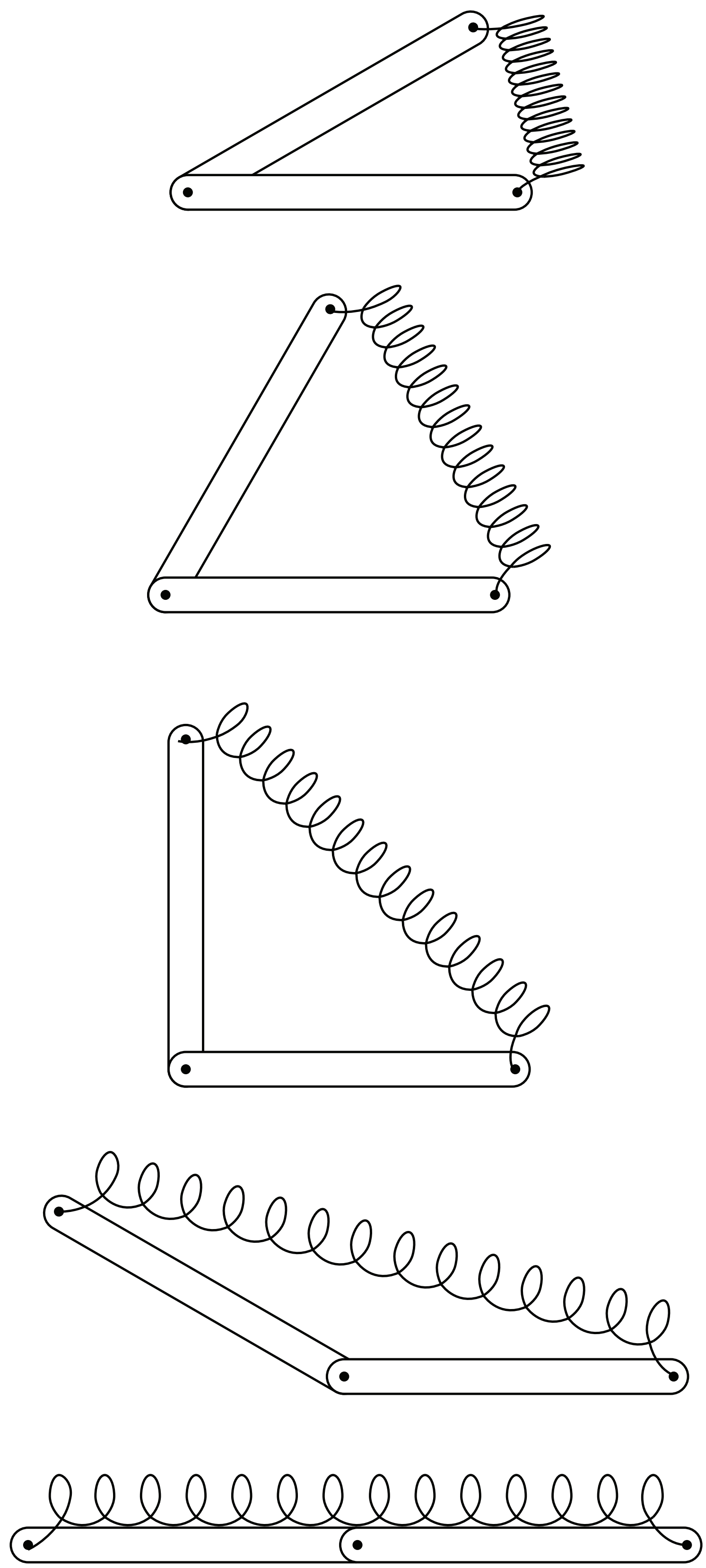
Chapter 3
The theorem of Pythagoras

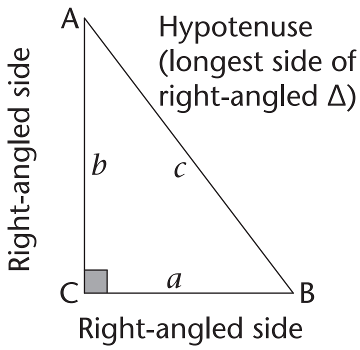
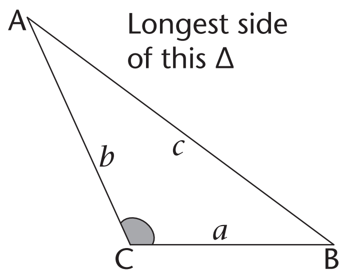
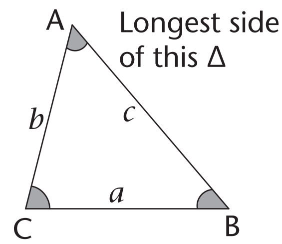
Right-angled triangles have a special feature that does not apply to other types of triangles. In this chapter, you will investigate this feature, which has come to be known as the theorem of Pythagoras. A theorem is a statement that is proved to be true through reasoning. Once you understand the theorem, you will practise applying it in various ways.



3 The theorem of Pythagoras

3.1 The lengths of sides of right-angled triangles

WHAT DO YOU REMEMBER ABOUT TRIANGLES?

Right-angled triangle (Δ) **One angle is 90°.**	**Obtuse-angled triangle (Δ)** **One angle is obtuse** **(between 90° and 180°).**	**Acute-angled triangle (Δ)** **All angles are acute** **(less than 90°).**
		

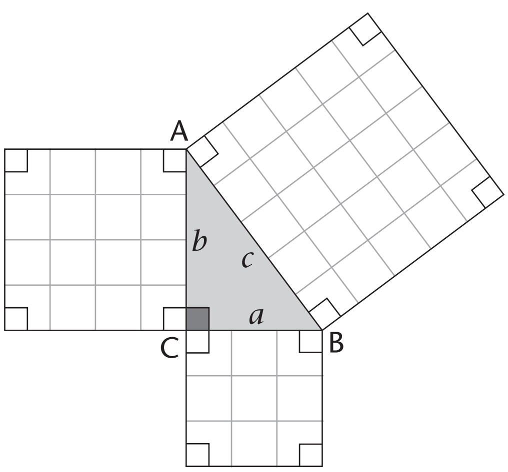
If the vertices of a triangle are labelled A, B and C, the sides opposite these vertices are often labelled as a, b and c, as shown in the above diagrams.

We use the word **hypotenuse** to indicate the side opposite the 90° angle of a right-angled triangle. The hypotenuse is always the longest side of a right-angled triangle. A triangle with no right angle does not have a hypotenuse.

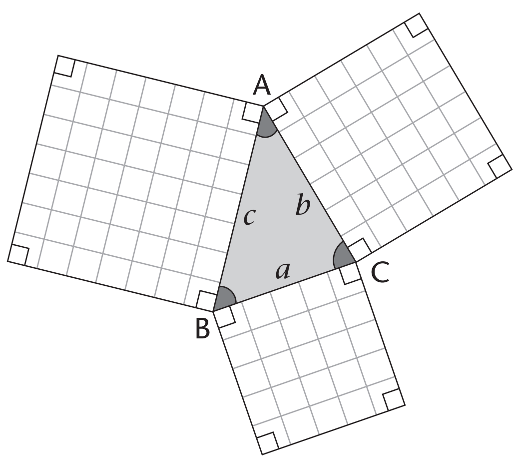
INVESTIGATING THE RELATIONSHIP BETWEEN THE LENGTHS OF SIDES

1. Study the figures below. Each triangle in the following four figures has a square drawn on each of its sides. So, in figure (a), $a = 3$ units, $b = 4$ units and $c = 5$ units long.

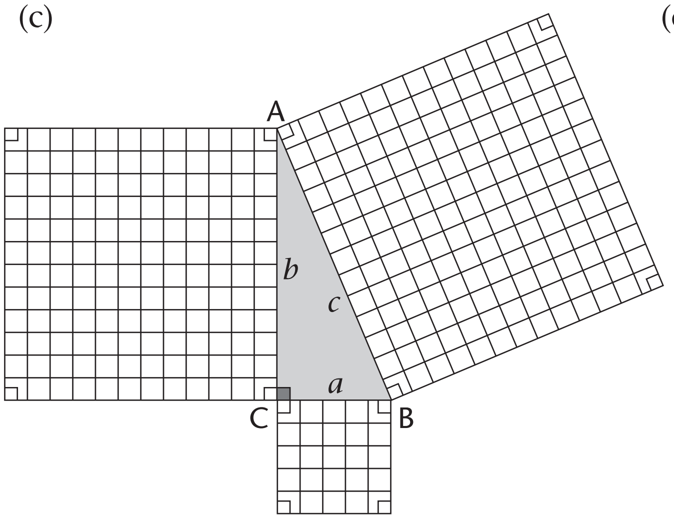
(a)



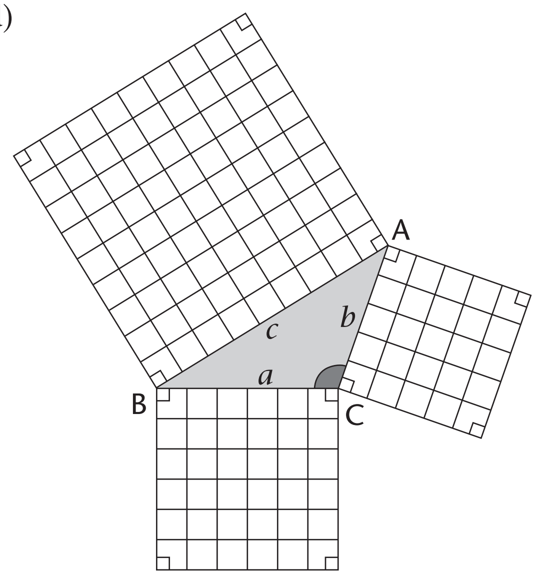
(b)



(c)



(d)



2. Refer to the four figures above to complete the table.

Figure	Type of triangle	Length of side *a*	Length of side *b*	Length of side *c*	a^2	b^2	c^2
(a)							
(b)							
(c)							
(d)							

3. Look at your completed table and then insert =, > or < in the following statements.

$a^2 + b^2$ ☐ c^2 when ΔABC is an acute-angled triangle.

$a^2 + b^2$ ☐ c^2 when ΔABC is an obtuse-angled triangle.

$a^2 + b^2$ ☐ c^2 when ΔABC is a right-angled triangle.

4. Which of the statements below are correct? ....

   A. In any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

   B. If a triangle is acute-angled, then the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.

   C. If a triangle is right-angled, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

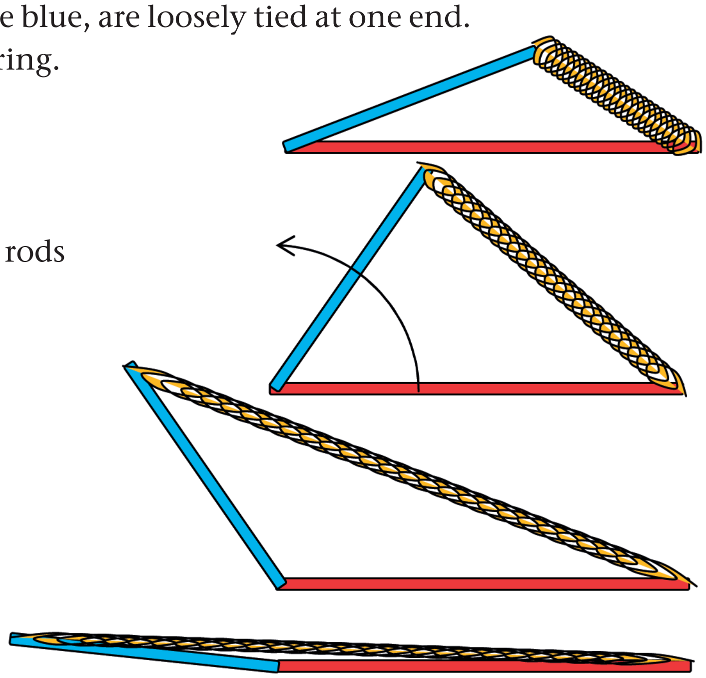
   D. In any obtuse-angled triangle, the area of the square on the longest side is equal to the sum of the area of the squares on the other two sides.

5. The following table gives the side lengths *a*, *b* and *c* of 10 triangles. Complete the table to decide what type of triangle each triangle is (acute-angled, obtuse-angled or right-angled).

a	b	c	$a^2 + b^2$	c^2	Fill in =, < or >	Type of triangle
7	8	10	$7^2 + 8^2 = 113$	$10^2 = 100$	$a^2 + b^2 > c^2$	Acute-angled
4	5	8	$4^2 + 5^2 = 41$	$8^2 = 64$	$a^2 + b^2 < c^2$	Obtuse-angled
6	8	10	$6^2 + 8^2 = 100$		$a^2 + b^2 = c^2$	Right-angled
8	13	17			$a^2 + b^2 \quad c^2$	
3	4	5			$a^2 + b^2 \quad c^2$	
5	6	7			$a^2 + b^2 \quad c^2$	
5	12	13			$a^2 + b^2 \quad c^2$	
15	8	17			$a^2 + b^2 \quad c^2$	
11	60	61			$a^2 + b^2 \quad c^2$	
12	35	37			$a^2 + b^2 \quad c^2$	

6. Two pieces of wood, one red and one blue, are loosely tied at one end. The two free ends are linked by a spring.

   The angle between the two wooden rods can be changed.



   Describe how this angle affects the length of the spring.

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3.2 Working with the theorem of Pythagoras

The special relationship between the lengths of the sides of a right-angled triangle is known as the **theorem of Pythagoras**. It can be stated in terms of area as follows:

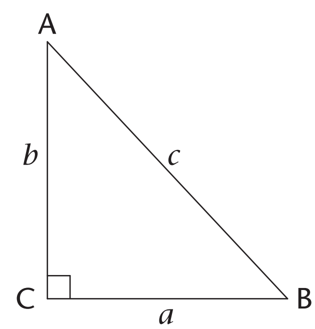
> If a triangle has a right angle, then the area of the square with a side on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

The reference to area can be left out.

> If a triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

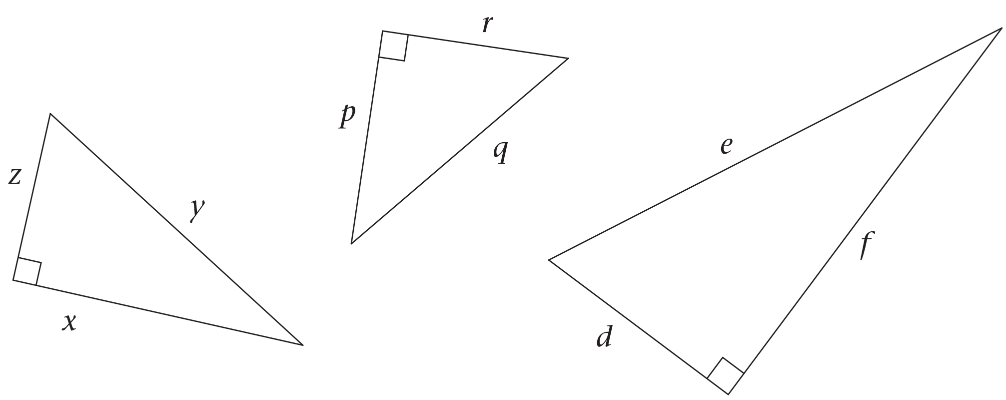
We can express the relationship between the lengths of the sides of the triangle by means of the equation $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides.

A note about Pythagoras
Pythagoras lived in about 500 BCE. The theorem is named after Pythagoras because he may have been the first person to prove it. However, the theorem was known and used in other parts of the world such as Egypt 1 200 years before Pythagoras was born.



WORKING WITH THE FORMULA

1. Write a Pythagorean equation for each of the following triangles. Explain what each letter symbol represents.



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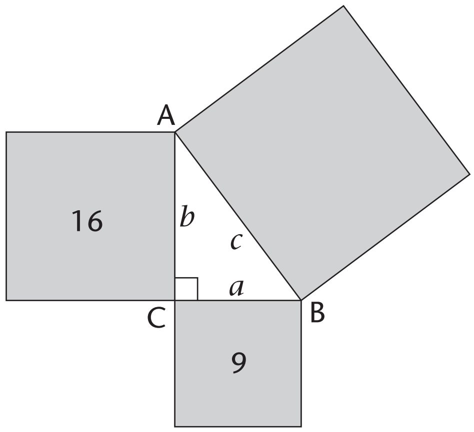
2. Study the worked example below.

Example

Consider the triangle below. Side a is 3 units long and side b is 4 units long. What is the length of side c?



If side a is 3 units long, and side b is 4 units long, then, according to Pythagoras' theorem:

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

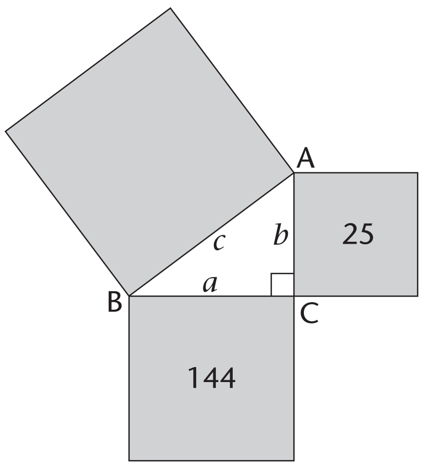
$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$

$c = 5$ units

3. The areas of some of the squares below are given. Calculate the areas of each of the squares that are not given and the lengths of all the sides.

(a)



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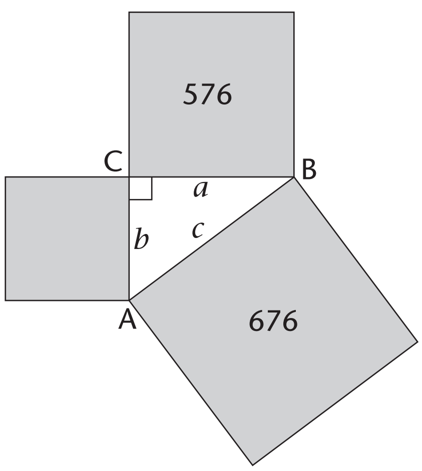
(b)



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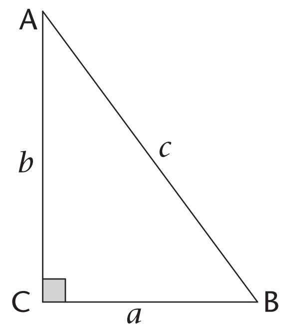
4. The following table gives information about the sides of five right-angled triangles. The letter symbol c represents the length of the hypotenuse in all cases. Use Pythagoras' theorem to complete the table, leaving answers in surd form if necessary.



a	b	c	a^2	b^2	$a^2 + b^2$	c^2
7	24					
16		34				
10				576		
			16	49		
	1		1			

3.3 Finding the missing sides in right-angled triangles

We can use the theorem of Pythagoras to calculate the length of the third side of a right-angled triangle if we know the lengths of the other two sides.

Example 1

A right-angled triangle has side $a = 6$ units and side $b = 8$ units. Calculate the length of side c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100 \\ \sqrt{c^2} &= \sqrt{100} \\ c &= 10 \\ \therefore c &= 10 \text{ units} \end{aligned}$$

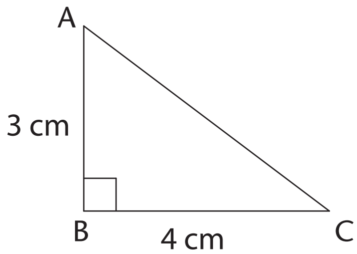
Example 2

A right-angled triangle has side $a = 5$ units and side $b = 3$ units. Calculate the length of side c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 3^2 \\ &= 25 + 9 \\ &= 34 \\ \sqrt{c^2} &= \sqrt{34} \\ c &= \sqrt{34} \text{ (leave in surd form)} \\ \therefore c &= \sqrt{34} \text{ units} \end{aligned}$$

CALCULATING THE LENGTH OF THE HYPOTENUSE

Use the formula for the theorem of Pythagoras to calculate the length of the hypotenuse. Leave answers in surd form if necessary.

1. 

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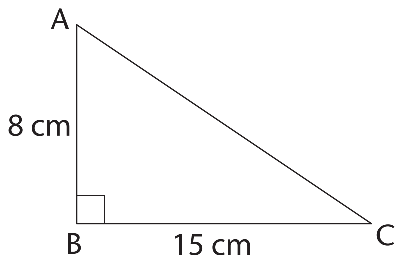
2. 

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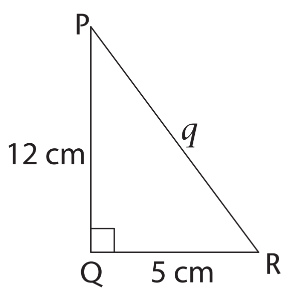
3. 

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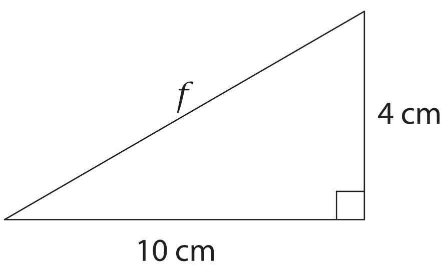
4. 

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5. A right-angled triangle with hypotenuse c and sides the following lengths:
$a = 9$ cm, $b = 40$ cm.

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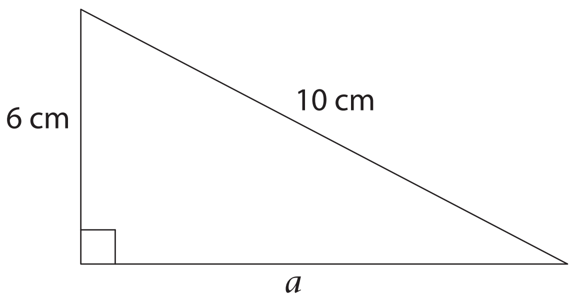
CALCULATING THE LENGTH OF ANY SIDE IN A RIGHT-ANGLED TRIANGLE

Calculate the missing sides in the following triangles. Do not use a calculator and leave the answers in the simplest surd form where necessary.

1.



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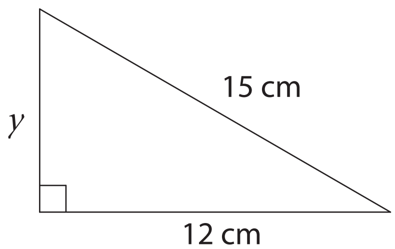

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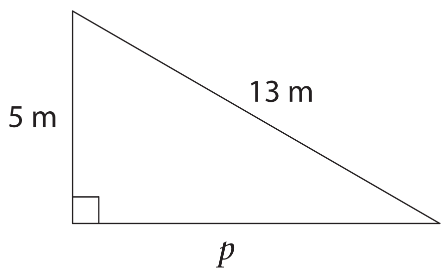

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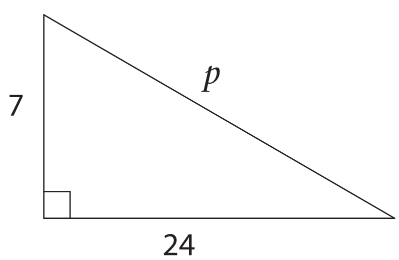

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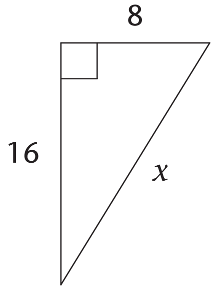

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3.4 Are the triangles right-angled?

You learnt in sections 3.1 and 3.2 that in a right-angled triangle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

How can we tell whether a triangle is right-angled if we are given the lengths of the sides? One way is to use the "converse" of the Pythagoras theorem.

> The converse states that if the sum of the squares of the lengths of two sides equals the square of the length of the longest side, then the triangle is a right-angled triangle.

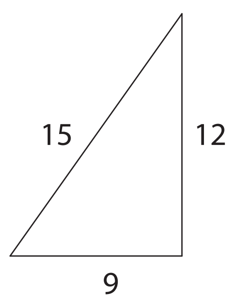
A converse is a statement that swaps around **what is given** in a theorem and **what must be determined**.

We can also state the converse as follows:

> If a triangle has side lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

In the questions that follow, you have to determine whether triangles are right-angled or not. You may study the example first.

Example: Determine whether the triangle is right-angled or not.

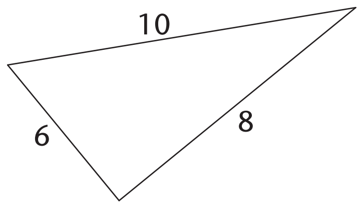


(Length of longest side)^2 = $(15)^2 = 225$
Sum of the squares of the lengths of the other two sides
$= 9^2 + 12^2$
$= 81 + 144$
$= 225$
(Longest side length)^2 = Sum of squares of other two sides lengths
And this can be written as $15^2 = 9^2 + 12^2$
$\therefore$ The triangle is right-angled.

RIGHT-ANGLED OR NOT?

Determine whether the triangles are right-angled or not.

1.



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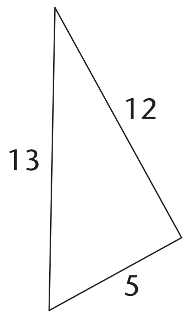
2.



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3. A triangle has sides measuring 6, 9 and 15 units.

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4. Which of the following lengths of sides of a triangle will form a right-angled triangle? Answer without doing any calculations and explain your answer.

(a) 4, 2, 2 (b) 6, 8, 10 (c) 9, 12, 15

(d) 3, 4, 6 (e) $3x$, $4x$, $5x$ (f) 30, 40, 50

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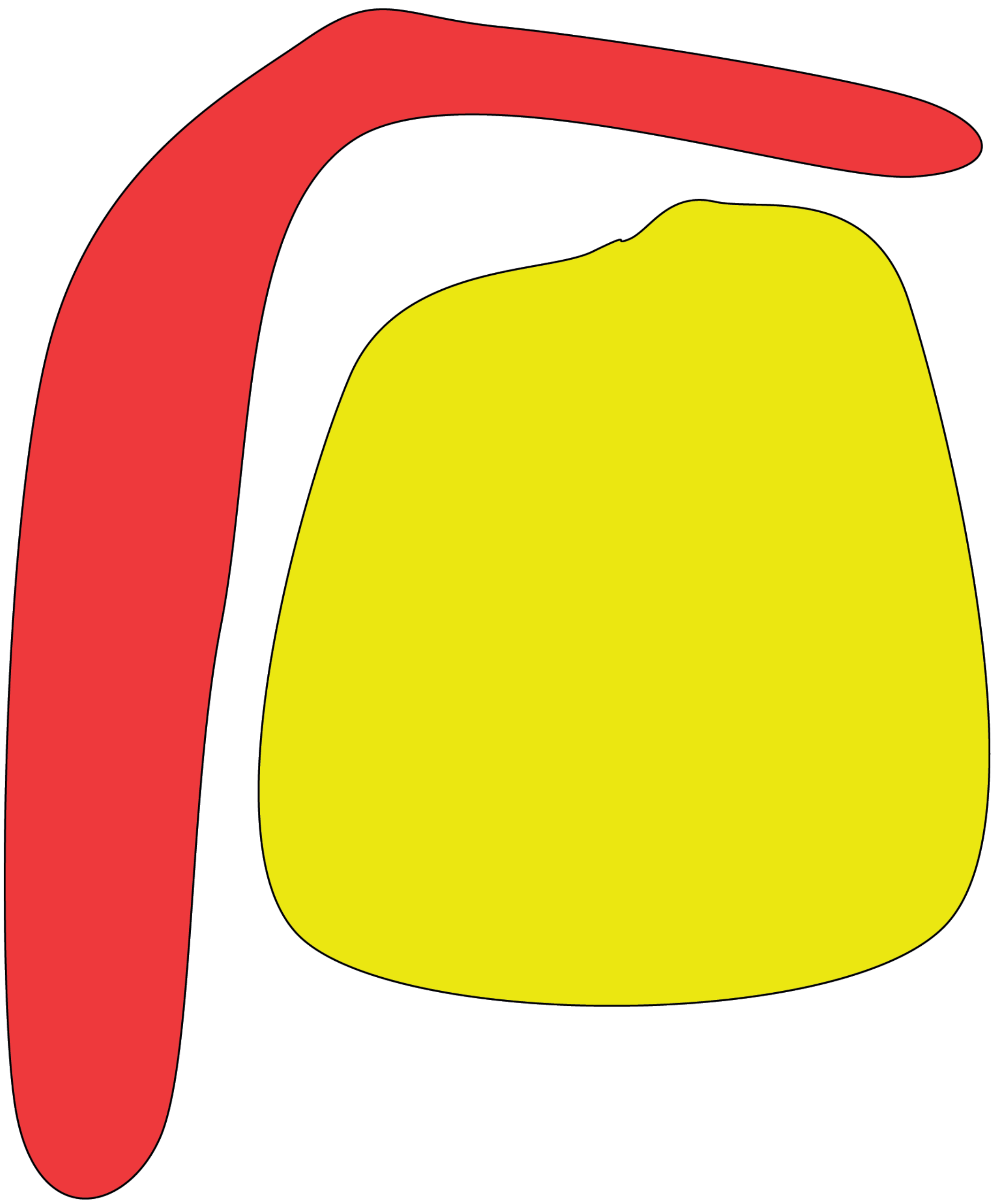
Chapter 4
Perimeter and area of 2D shapes

In Grade 7, you learnt to use formulae to calculate the perimeter (distance around a figure) of squares and rectangles, and the area (size of the flat surface) of squares, rectangles and triangles. In this chapter, you will revise the formulae you learnt, and you will investigate and use formulae to calculate the perimeter and area of circles. This chapter also includes some practice in converting between different units that we use to measure area, namely square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).



Which is bigger, the red shape or the yellow shape?

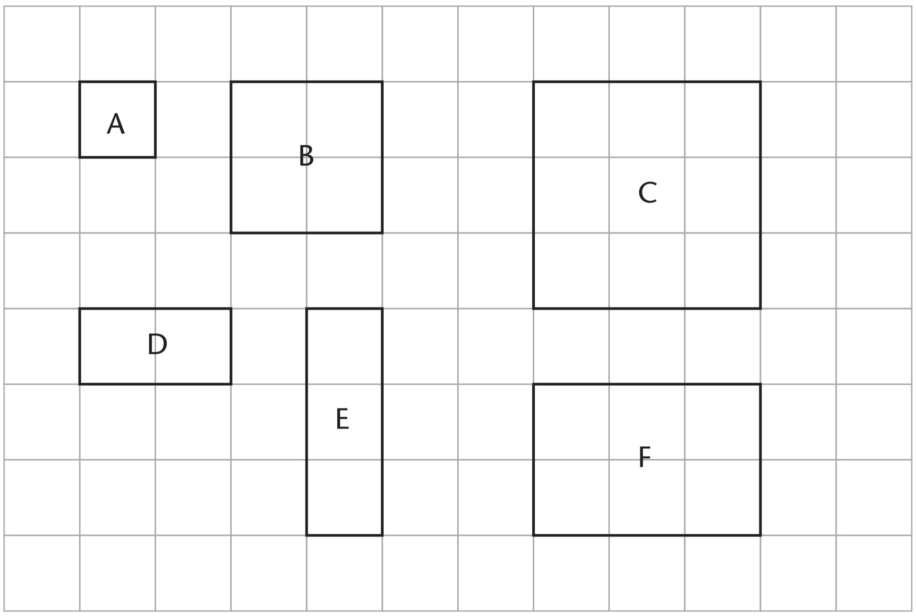
4 Perimeter and area of 2D shapes

4.1 Perimeter of squares and rectangles

The **perimeter** (*P*) of a flat shape is the distance around a shape. We measure it in units such as millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

EXPLAINING THE FORMULAE FOR PERIMETER

1. Each block in the grid below measures 1 cm × 1 cm. Calculate the perimeter of each shape by adding up the lengths and breadths.



Shape	A	B	C	D	E	F
Length						
Breadth						
Perimeter						

2. Explain to a partner why the following formulae for perimeter are correct.
   Perimeter of a square = $4s$ or (4 × length of a side)
   Perimeter of a rectangle = $2(l + b)$ or $2l + 2b$ (l is the length and b is the breadth)

3. Use the formulae in question 2 to calculate the perimeters of shapes A to F above.

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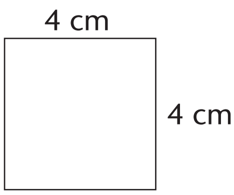
CALCULATING PERIMETERS USING FORMULAE

Use formulae to calculate the perimeters of the following shapes.

1.



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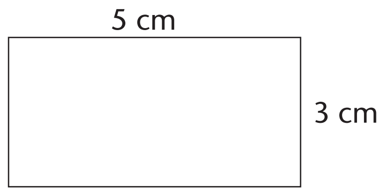

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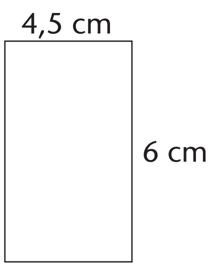

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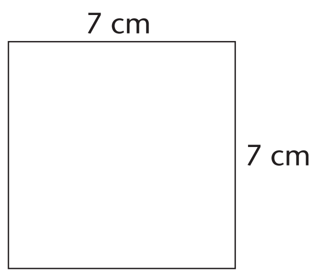

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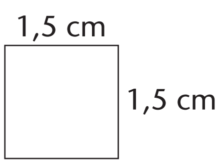
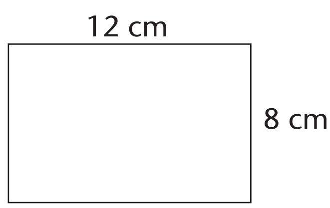


6.

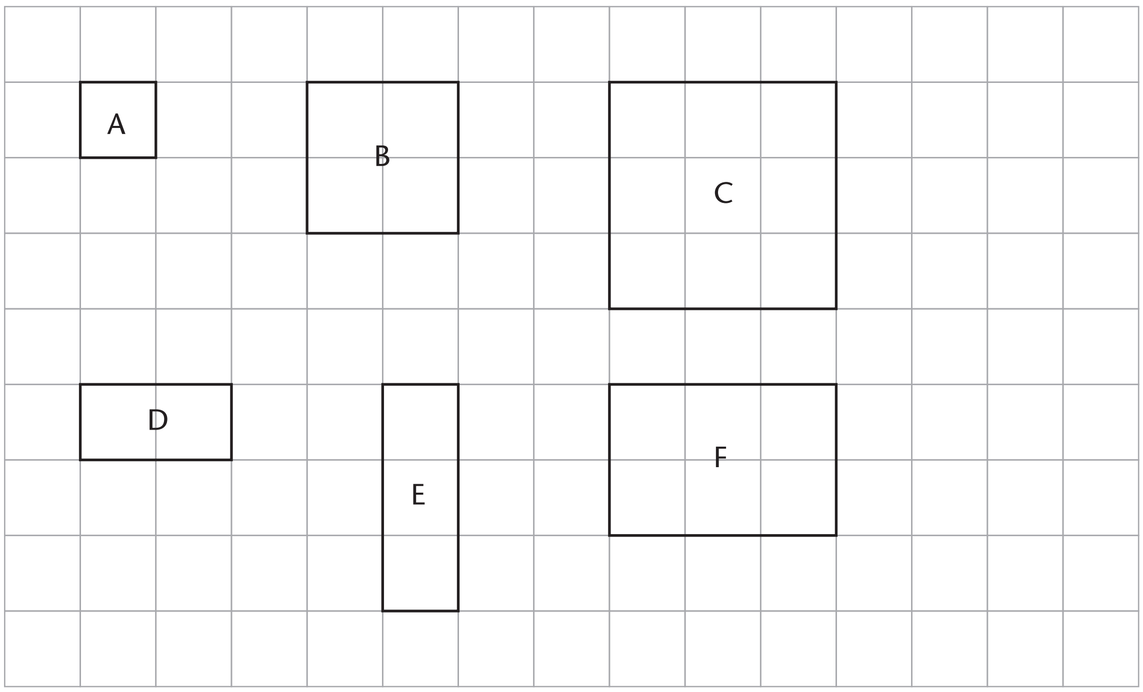


4.2 Area of polygons

We use square units such as mm^2, cm^2, m^2 and km^2 to measure the **area** (A), or the size of a flat surface, of a shape.

AREA OF SQUARES AND RECTANGLES

1. How many square units make up the area of the following shapes? Write the answers below or next to the shapes.



2. Each square on the grid above measures 1 cm × 1 cm (or 1 cm^2). Write down the area of each shape above in square centimetres (cm^2).

....

Below are formulae for calculating area:

Area of a square = s^2

Area of a rectangle = $l \times b$

3. Calculate the areas of shapes C, E and F in question 1 using the formulae.

....

....

....

SOLVING MORE PERIMETER AND AREA PROBLEMS

1. The perimeter of a square is 8 cm. What is the length of each side?

2. The area of a rectangle is 40 cm^2 and its length is 8 cm. What is its breadth?

3. The perimeter of a square is 32 cm. What is its length and area?

4. The area of a rectangle is 60 cm^2 and its length is 12 cm. What is its breadth and perimeter?

5. A rectangular yard has an area of 600 m^2. If the breadth is 20 m, find the length and the perimeter.

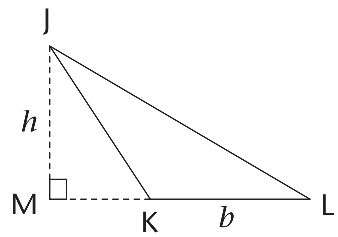
6. A square has an area of 10 000 m^2. What is the perimeter?

AREA OF TRIANGLES

In Grade 7 you learnt how to calculate the area of a triangle with the following formula:

> Area of a triangle = $\frac{1}{2}$(base × perpendicular height) = $\frac{1}{2}(b \times h)$

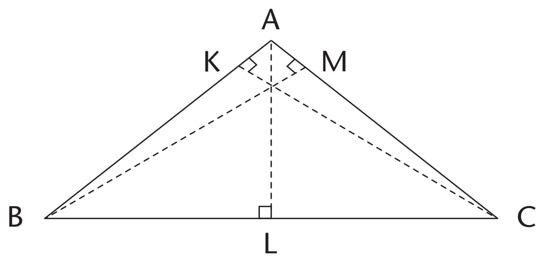
Any of the three sides of a triangle can be regarded as the **base**. The shortest distance from the vertex opposite the chosen base to the base is called the **height** of the triangle with respect to the chosen base. If the triangle is obtuse angled, the line showing the height is outside the triangle. For example, in ΔJKL, JM is the height with respect to the base KL.

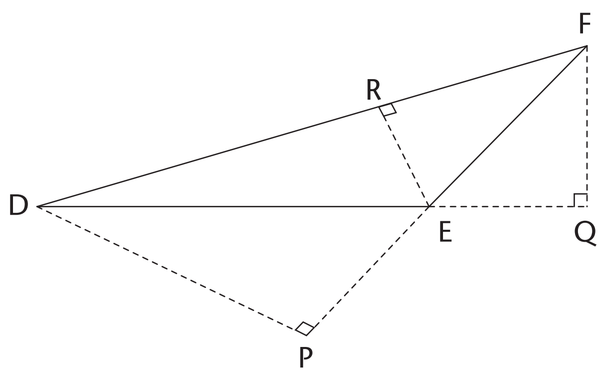


To calculate the area of a triangle with the above formula, the height with respect to the chosen base must be used.

PROBLEMS INVOLVING THE AREA OF TRIANGLES

1. Complete the table below by writing down the name of each base and its matching height in ΔABC and ΔDEF:

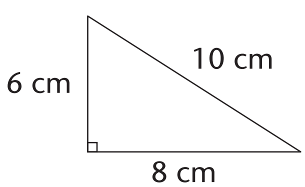




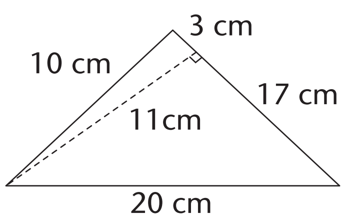
Base						
Height						

2. Calculate the area of the following triangles.

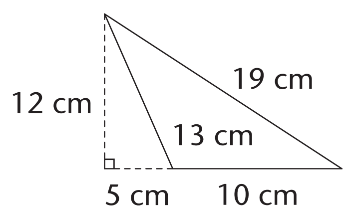
(a)



(b)



(c)



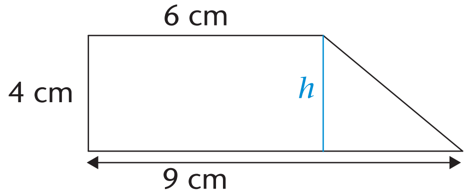
Maths2_Gr8_LB_Book.indb 59 2014/09/04 10:39:10 AM

AREA OF COMPOSITE SHAPES

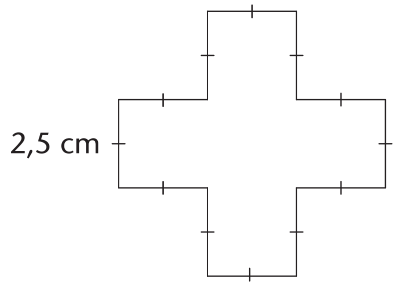
A **composite shape** is made up of a number of other shapes. Often, we can break up the shape into rectangles, squares or triangles to help us work out the area of the shape.

1. Use a ruler and pencil to divide each of the following shapes into rectangles, squares and/or triangles. The first one has been done for you.
2. Work out the length of the sides you need and then calculate the area of the shapes. Round off your answers to two decimal places where necessary.

(a)



(b)



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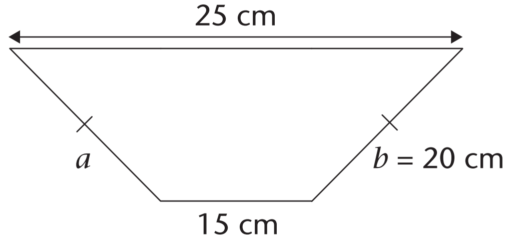
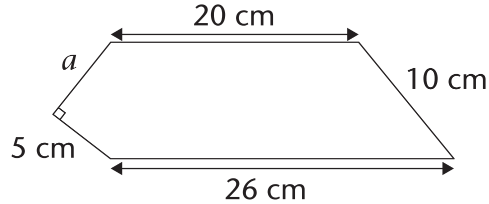
(c)



(d)



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4.3 Perimeter of circles

PARTS OF A CIRCLE

In Grade 7, you learnt about the different parts of a circle, including the following:

The **centre** of a circle is the point in the middle (centre) of the circle.

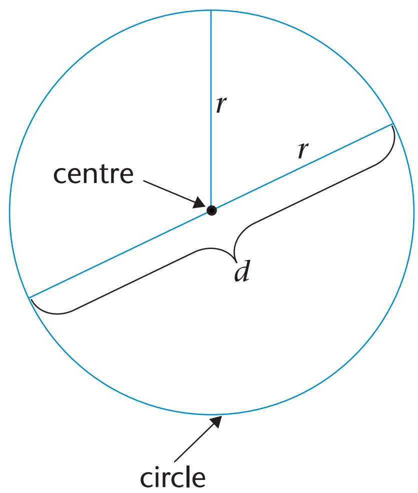
The **circumference** (*C*) is the distance around the circle. It is the length of the curved line that forms the circle.

The **radius** (*r*) is the line segment drawn from the centre of the circle to any point on the circle.

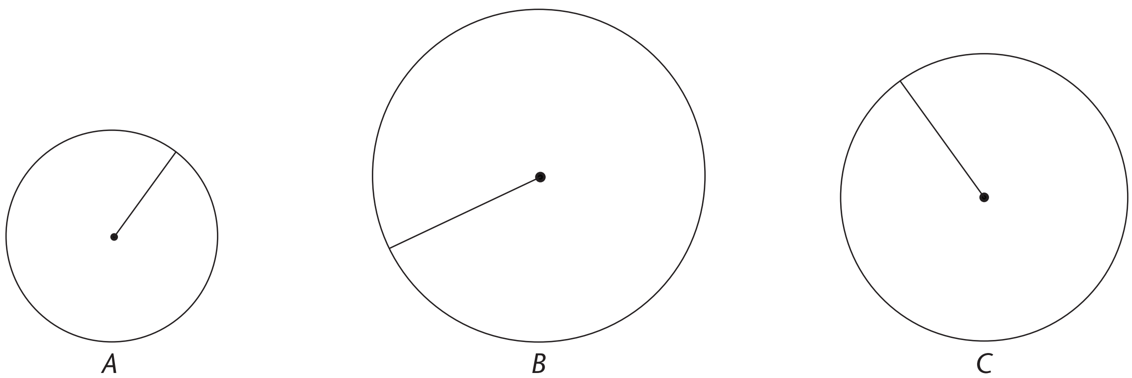
The **diameter** (*d*) is the line segment passing through the centre of the circle and joining any two points on the circle.

The length of the radius is always half the length of the diameter: $r = \frac{1}{2}d$

The length of the diameter is always twice the length of the radius: $d = 2r$



1. Use a ruler to measure the radii (plural of radius) given below and then write down the lengths of both the radii and diameters of the circles in the table below.



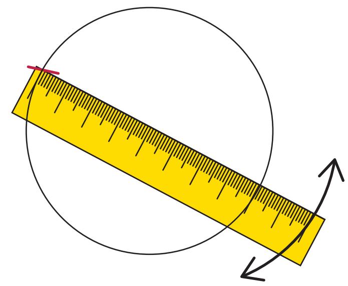
Circle	A	B	C
Radius (mm)			
Diameter (mm)			

2. Write down the diameters of circles with the following radii:

(a) $r = 8$ cm (b) $r = 1$ m (c) $r = 4{,}5$ cm (d) $r = 6{,}2$ m

....

RELATIONSHIP BETWEEN A CIRCLE'S CIRCUMFERENCE AND DIAMETER

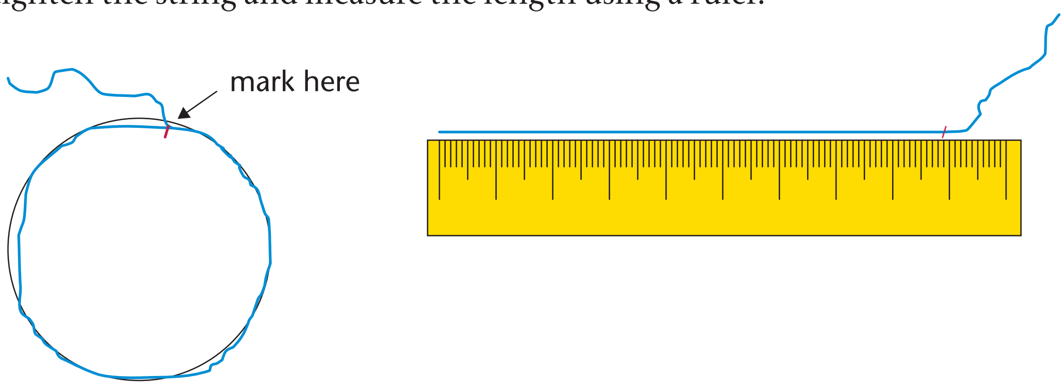


If you do not know where the **centre** of a circle is, you can determine it by measuring the diameter as follows:

- Mark a point on the circle from which to measure.
- Keeping the '0' of the ruler in place, move the other end of the ruler until you find the longest distance. This is the diameter.

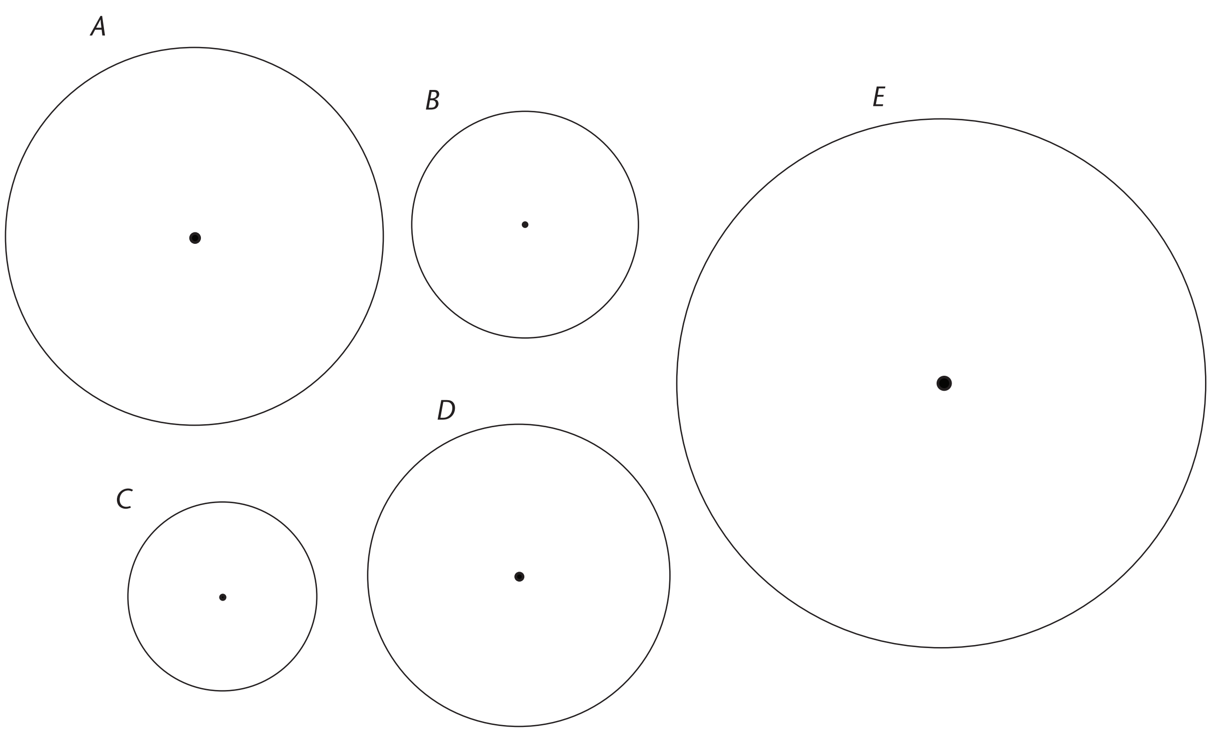
You can get a rough measurement of the **circumference** of a circle as follows:

- Use a string and lay it around the edge of the circle as accurately as possible.
- Mark the string when you reach the point where you first started measuring.
- Straighten the string and measure the length using a ruler.



Circles of different sizes are given below. The circumferences are shown in the table in question 2 on the next page, rounded off to two decimal places.

1. Measure the diameter of each circle and write it in the table.



2. Use a calculator to work out the answers in the last column. (Round off to two decimal places.)

Circle	Diameter (cm)	Circumference (cm)	Circumference ÷ diameter
A		15,71	
B		9,42	
C		7,85	
D		12,57	
E		21,99	

3. What do you notice?

....

....

PI (π) AND THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

In the previous activity, you should have found that the circumference of a circle divided by its diameter is always equal to the same number. This number is a constant value and is called **pi**. *Pi* is a Greek letter and its symbol is π.

You also worked with values rounded off to two decimal places (hundredths). But actually, π is an **irrational number**. This means that the numbers after the decimal comma go on and on without ending and without repeating. On a calculator, you will find that the value for π is given as 3,141592654 (correct to 9 decimal places).

When we use π in our calculations, we usually round it off as $\pi \approx \frac{22}{7}$ or 3,14.

In the previous activity, you found that, for any circle, $\frac{C}{d} = \pi$ (the circumference divided by its diameter is equal to the constant, π). Therefore, if we multiply the diameter of a circle by π, we should get the circumference of the circle:

$$\begin{aligned}\text{Circumference of a circle } (C) &= \pi d \\ &= \pi(2r) \\ &= 2\pi r\end{aligned}$$

USING THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

In the following calculations, use $\pi = 3{,}14$ and round off your answers to two decimal places where necessary.

1. Calculate the circumference of a circle with:
   (a) a radius of 2 cm
   (b) a radius of 10 mm
   (c) a diameter of 8 cm
   (d) a diameter of 25 mm
   (e) a radius of 40 m
   (f) a diameter of 100 m

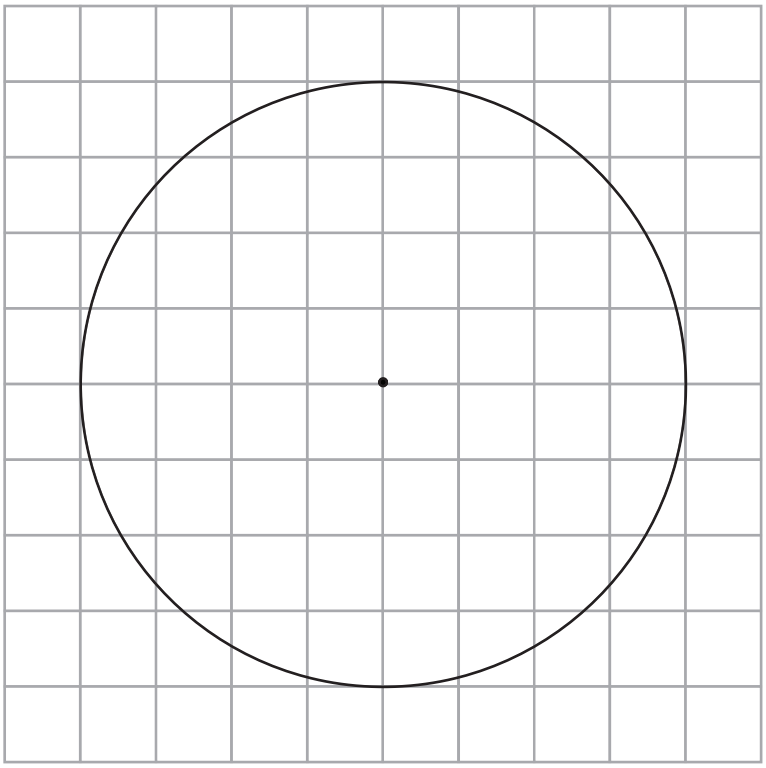
2. Calculate the radius and circumference of a circle with a diameter of:
   (a) 125 mm
   (b) 70 cm

3. Calculate the radius of a circle with a circumference of:
   (a) 110 cm
   (b) 200 m

4.4 Area of circles

INVESTIGATING THE FORMULA FOR THE AREA OF A CIRCLE

1. Each square in the grid below measures 1 cm by 1 cm (1 cm²).



(a) Count the number of squares inside the circle. Estimate what the parts of squares add up to. What is the area inside the circle?

....

(b) What is the radius (*r*) of the circle?

....

(c) How accurate is the above method for finding the area of a circle?

....

....

(d) How can we improve on this method of using squares to approximate the area of a circle?

....

(e) Suppose instead of using 1 cm by 1 cm squares we use 0,5 cm by 0,5 cm squares to measure the area of the circle above. Which of the two measurements of area will be more accurate? Explain.

....

....

(f) Now suppose we use squares that are 0,25 cm by 0,25 cm. Which measurement will be the best estimate of the three?

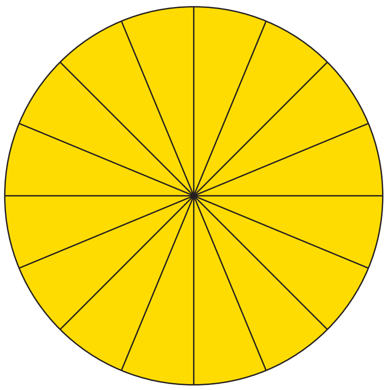
....

We can estimate area by placing a square grid over the surface of which we want to estimate the area. We can then count approximately how many squares are needed to cover the surface we wish to measure.

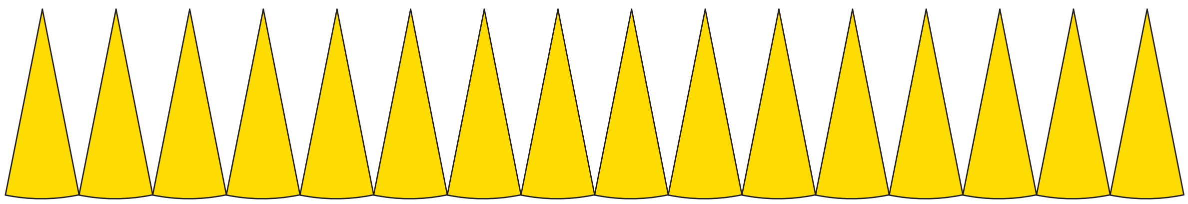
In the case of a curved surface like a circle the area cannot be accurately determined in this way; it can only be estimated. The accuracy of the estimate depends on the size of the squares used.

In this activity we are going to develop a formula for calculating the area of a circle.

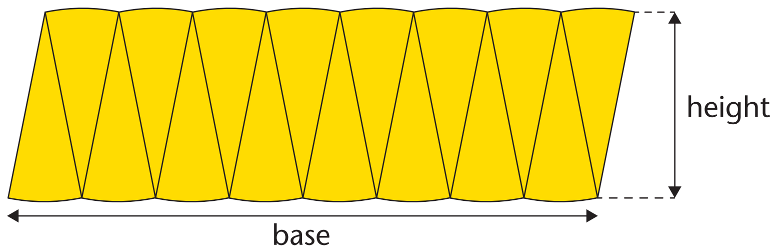
Consider the circle alongside. It has been divided into 16 identical sectors. We will use a technique that mathematicians sometimes use to transform a shape into one that they know something about in order to solve a problem.



The challenge here is that we want to find a way to calculate the area of a circle. We know how to find the area of a rectangle. Is there a way that we can redraw a circle so that it looks something like a rectangle? One way to go about this is to divide the circle into 16 identical sectors. We then cut the circle into 16 different pieces as shown below.



We then re-arrange the sectors like this.



2. We have transformed the circle by cutting it into identical sectors and re-arranging them. What does this shape look like?

....

3. What does the
   (a) height of the shape above match in the original circle? ....
   (b) base of the shape match in the original circle? ....

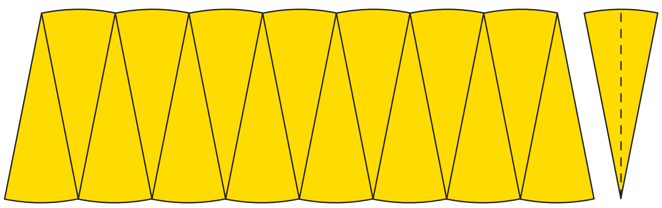
4. Is there a way in which we can make the challenge easier for ourselves?

....

....

5. The last sector in the arrangement below is further divided in half.
   (a) What shapes are formed from dividing the sector?

....



   (b) What new shape will be formed by placing each half of the sector on either side of the shape above?

....

6. What does the
   (a) height of the new shape correspond to in the original circle?

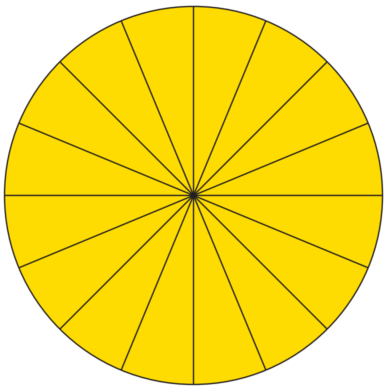
....

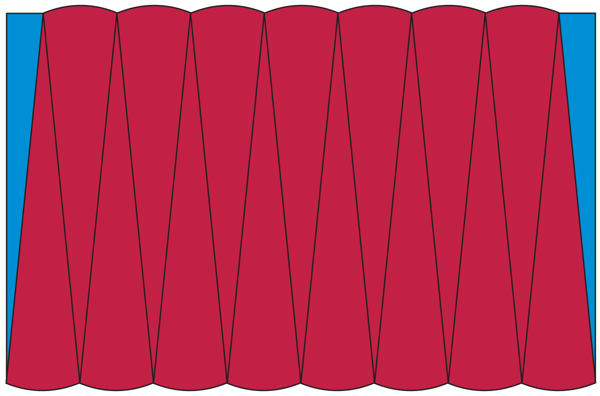
   (b) base of the new shape correspond to in the original circle?

....

Maths2_Gr8_LB_Book.indb 67 2014/09/04 10:39:12 AM

You have probably noticed that when we divide a circle into many sectors and then re-arrange the sectors, they form a rectangular shape. Try to make sense of the argument presented below.





$C = 2\pi r$

Area = $l \times b$

Area = $\frac{1}{2} \times 2 \times \pi \times r \times r$

Area = πr^2

7. (a) Use the formula $A = \pi r^2$ to calculate the area of a circle with a radius of 4 cm. Use $\pi = 3{,}14$.

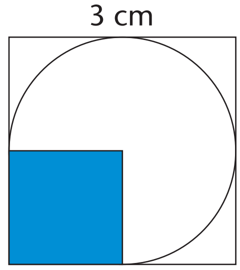
....

(b) How close is this answer to the number of squares you calculated inside the circle in question 1 on page 65?

....

From now onwards we will use the **formula** $A = \pi r^2$ to calculate the **area of a circle**, where r is the length of the radius. You will be given the value of π to use in the calculations. The value of π is usually given correct to 2 decimal places as 3,14.

8. How can we interpret r^2 in the formula $A = \pi r^2$? Use the figure on the right to answer the questions below:



(a) What is the radius of the circle? ....

(b) The length of the blue square is 1,5 cm. What is its area?

....

(c) What is the value of r^2? ....

(d) If r is the radius of a circle, then r^2 is ....

USING THE FORMULA FOR THE AREA OF A CIRCLE

In the following calculations, use 3,14 as an approximation for π and round your answers off to two decimal places. Use a calculator where necessary.

1. Calculate the area of a circle with a radius of:

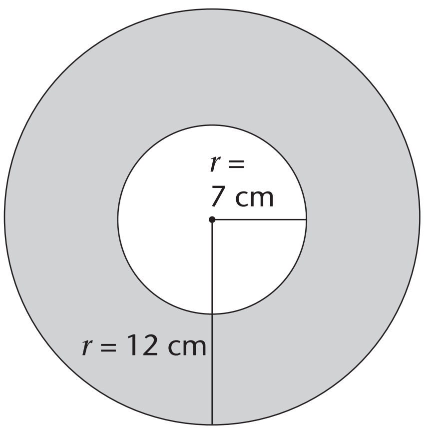
   (a) $r = 8$ cm (b) $r = 4{,}5$ cm

2. Calculate the radius of a circle with the following area:

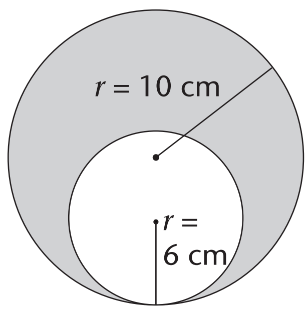
   (a) 100 m^2 (b) 76 m^2

3. Work out the area of the shaded parts of the following shapes:

   (a)



   (b)



4.5 Converting between square units

You already know how to convert between units we use to measure lengths or distances, for example mm, cm, m and km:

To convert	Do this	To convert	Do this
cm to mm	× 10	mm to cm	÷ 10
m to cm	× 100	cm to m	÷ 100
km to m	× 1 000	m to km	÷ 1 000

Use this knowledge to work out how to convert between square units (mm^2, cm^2, m^2 and km^2).

1. Convert cm^2 to mm^2
   $1\ cm^2 = 1\ cm \times 1\ cm$
   $= 10\ mm \times 10\ mm$
   = ...

2. Convert m^2 to cm^2
   $1\ m^2 = 1\ m \times 1\ m$
   = ... cm × .... cm
   = ...

3. Convert km^2 to m^2
   $1\ km^2$ = ... km × .... km
   = ...
   = ...

4. Convert mm^2 to cm^2
   $1\ mm^2 = 1\ mm \times 1\ mm$
   $= 0{,}1\ cm \times 0{,}1\ cm$
   = ...

5. Convert cm^2 to m^2
   $1\ cm^2$ = ... cm × .... cm
   = ...
   = ...

6. Convert m^2 to km^2
   $1\ m^2$ = ... m × .... m
   = ...
   = ...

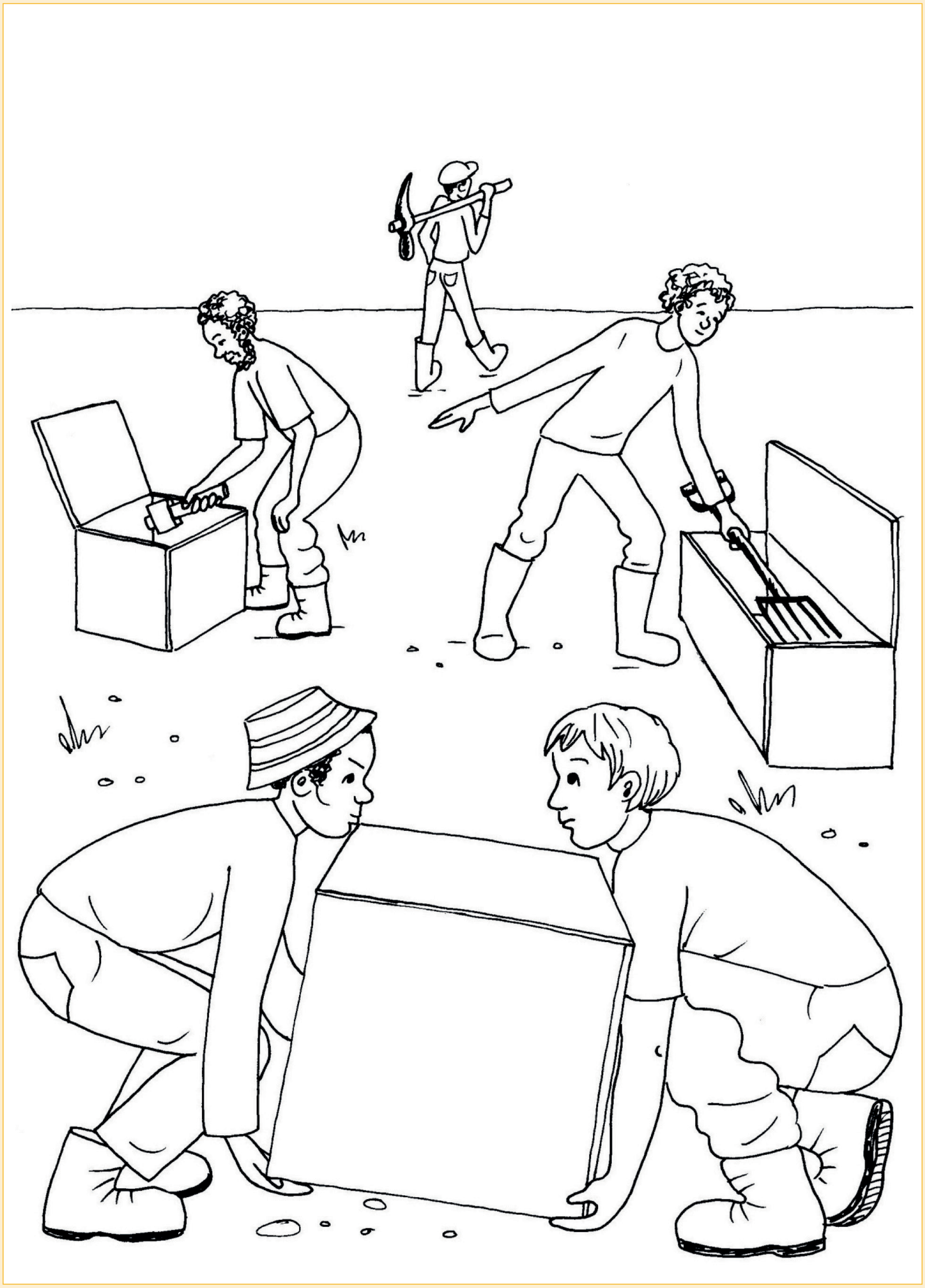
7. Complete the following table.

To convert	Do this	To convert	Do this
cm^2 to mm^2		mm^2 to cm^2	
m^2 to cm^2		cm^2 to m^2	
km^2 to m^2		m^2 to km^2	

Maths2_Gr8_LB_Book.indb 70 2014/09/04 10:39:12 AM

CHAPTER 5
Surface area and volume of 3D objects

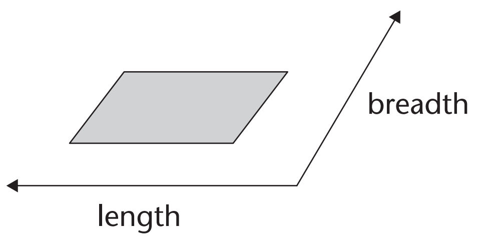
The surface area of an object is the size of the flat surfaces all around the object. The volume of an object is the amount of space that the object takes up. In this chapter, you will use formulae to calculate the volumes and surface areas of cubes, rectangular prisms and triangular prisms. You will also investigate the relationship between surface area and volume, as well as revise how to convert between the different units used to measure volume.



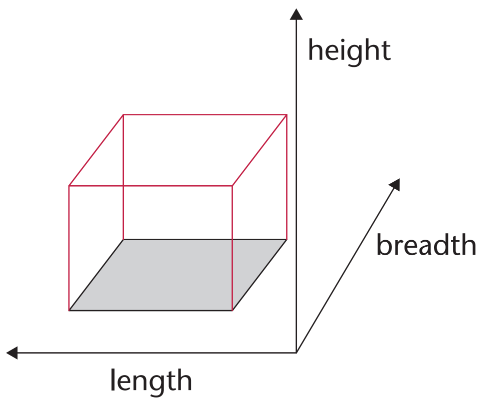
5 Surface area and volume of 3D objects

5.1 From 2D to 3D measurements

Remember that 2D shapes have only length and breadth, while 3D objects have length, breadth and height.



2D shape



3D object

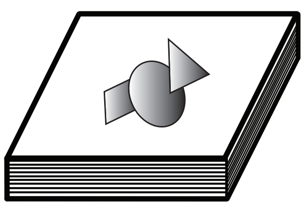
A 2D shape has only one surface. We call the size of this flat surface the **area** of the shape.

A 3D object has more than one surface. For example, a cube has 6 surfaces, or faces. The sizes of these surfaces on the outside of the 3D object are called its **surface area**.

A 2D shape is flat, so it takes up space in only two directions. But a 3D object has height as well, so it takes up space in a third direction also. The space that a 3D object takes up is called its **volume**.

INVESTIGATING THE SURFACE AREA AND VOLUME OF A BOOK

Work with a partner. Choose a book each. The books must be different sizes.



1. Run your hand over all the outside surfaces of your book. How many surfaces (or faces) does your book have?

   ....

2. Estimate whether the surface area of your book is bigger or smaller than that of your partner's book.

   ....

3. If you were to cover the book with wrapping paper, explain how you would calculate the minimum size of paper you would need.

....

....

....

4. Estimate whose book takes up the most space. How could you calculate which book really takes up the most space?

....

5.2 Surface area of 3D objects

USING NETS TO EXPLORE SURFACE AREA

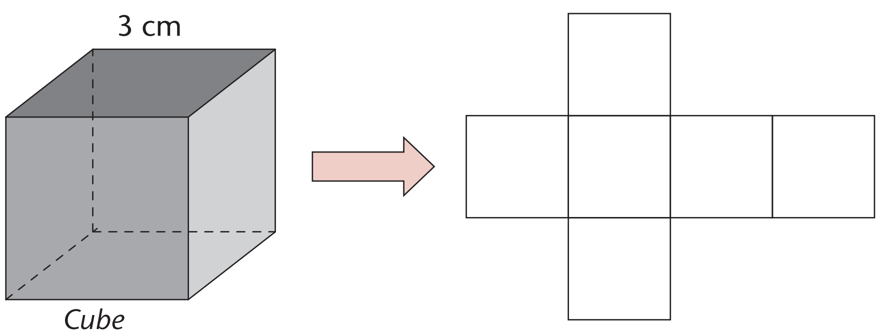
The **surface area** of an object is equal to the sum of the areas of all its faces. So we can use the net of an object to investigate its surface area.

A net is a flat shape that can be folded to make a 3D object.

The diagrams below show 3D objects with their matching nets.

1. Use the measurements given to calculate the area of each face shown by the net. (Use your calculator if necessary and round off to two decimal places.)
2. Add up the areas to calculate the surface area of each object.

A



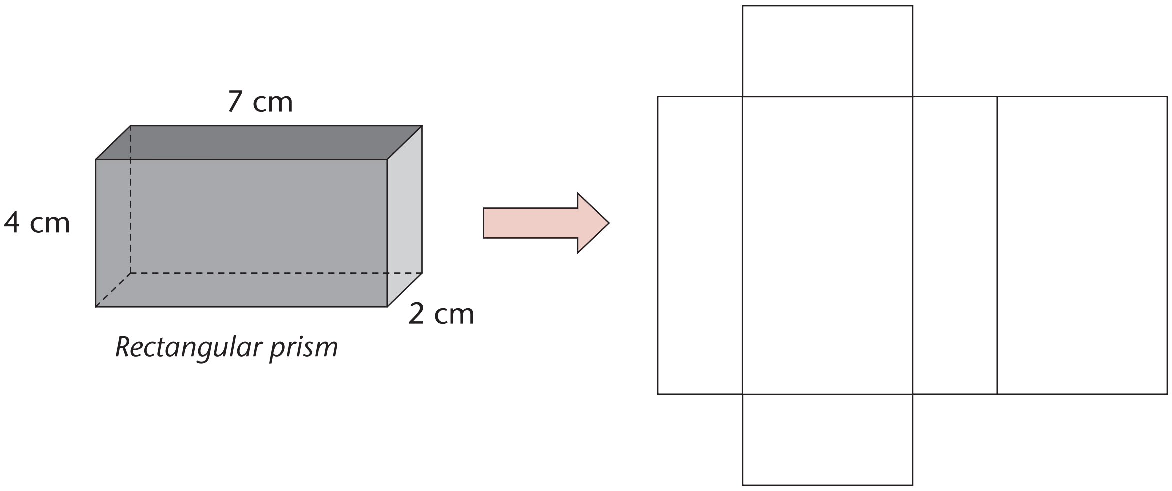
....

....

....

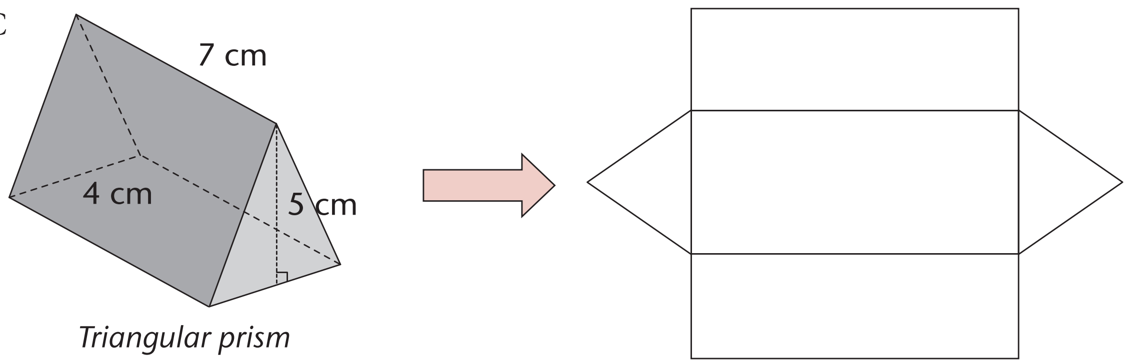
....

B



Rectangular prism

C



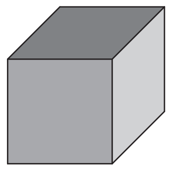
Triangular prism

DEDUCING FORMULAE FOR SURFACE AREAS

The surface area of a prism = the sum of the areas of all its faces

1. (a) Use the general formula above and the work you did on the cube on page 74 to determine which of the following formulae are correct. Tick the correct one(s).

   - ☐ Surface area of a cube = $4 \times s$
   - ☐ Surface area of a cube = $s \times s \times s \times s$
   - ☐ Surface area of a cube = $6 \times s^2$
   - ☐ Surface area of a cube = s^6



   (b) Explain your choice above.

   ....

2. (a) Write a formula for the surface area of any rectangular prism.

   ....

   ....

   (b) Explain your formula.

   ....

   ....

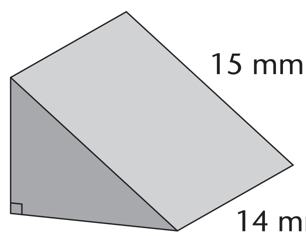
   ....

   ....

3. (a) Write a formula for the surface area of any triangular prism.

   ....

   ....



   (b) Explain your formula.

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4. Use the formulae in questions 1 to 3 to calculate the surface areas of the cube, rectangular prism and triangular prism shown in questions 1 to 3.
   Surface area of cube:

   ....
   ....
   ....

   Surface area of rectangular prism:

   ....
   ....
   ....
   ....

   Surface area of triangular prism:

   ....
   ....
   ....
   ....

SURFACE AREA CALCULATIONS

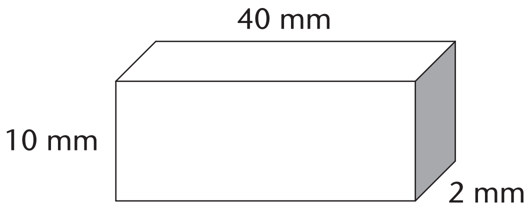
Work out the surface areas of the following four objects.
Give all answers in cm^2.

Remember:

$1\ cm^2 = 100\ mm^2$	$1\ mm^2 = 0{,}01\ cm^2$
$1\ m^2 = 10\ 000\ cm^2$	$1\ cm^2 = 0{,}0001\ m^2$
$1\ km^2 = 1\ 000\ 000\ m^2$	$1\ m^2 = 0{,}000001\ km^2$

It may be a good idea to sketch the net for each object before doing the calculations.

1.



   ....
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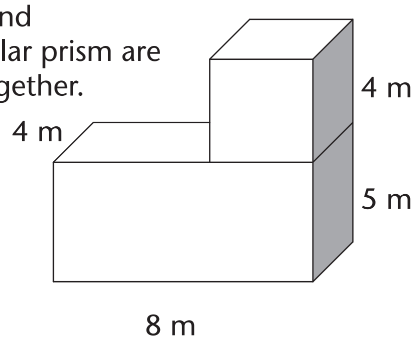
2.

A cube and rectangular prism are glued together.



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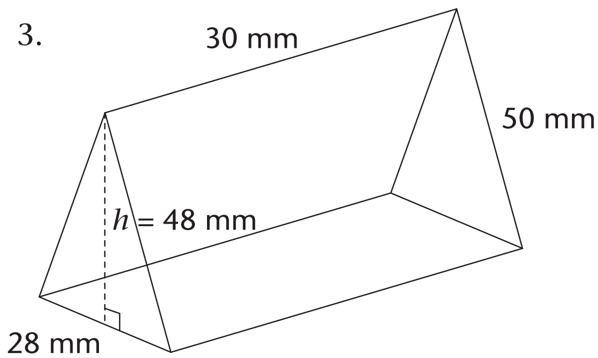
3.



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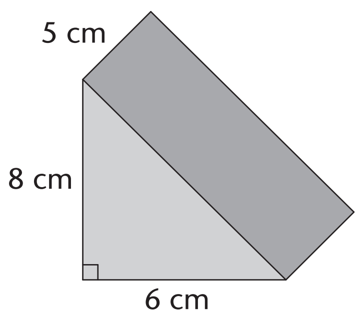
4.



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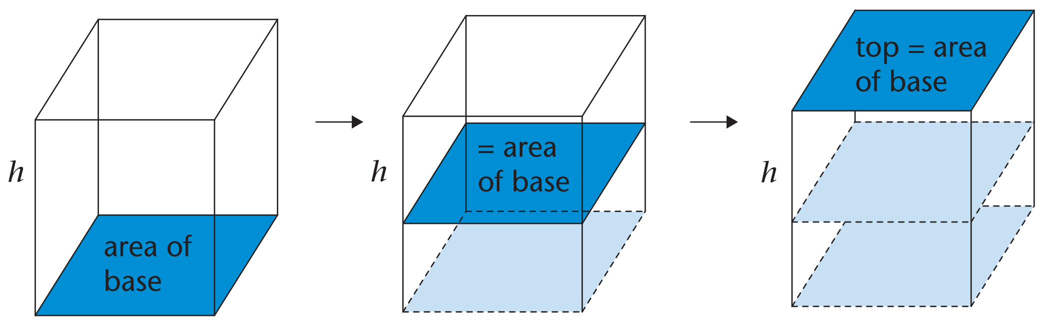
5.3 Volume of 3D objects

DERIVING FORMULAE TO CALCULATE VOLUME

Think of a prism and its base. If you were to move the base up to the top, between the lateral faces of the prism, the area of the base would remain exactly the same.

Lateral faces are faces that aren't bases.

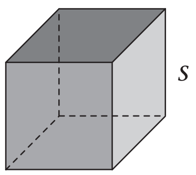


The volume of a prism = Area of base × height

Volume is the amount of space that an object takes up.

Use this general formula above to write the formula for the volume of a cube, a rectangular prism and a triangular prism.

A. Cube



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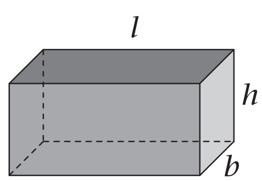
B. Rectangular prism



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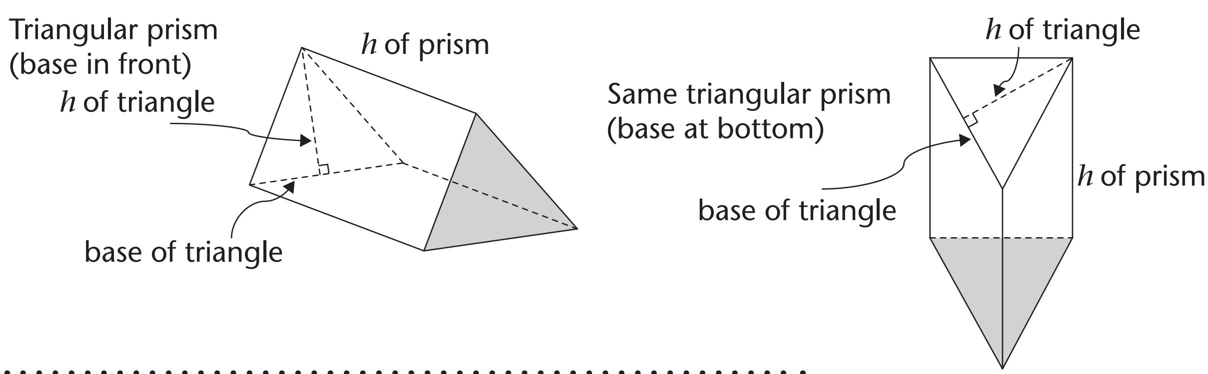
Note about triangular prism
Do not get confused between:
- the base of the prism and the base of the triangular face of the prism
- the height of the prism and the height of the triangular face of the prism.

C. Triangular prism



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Maths2_Gr8_LB_Book.indb 79 2014/09/04 10:39:15 AM

You should have found the following volume formulae:

Volume of a cube = s^3 or $s \times s \times s$

Volume of a rectangular prism = $l \times b \times h$

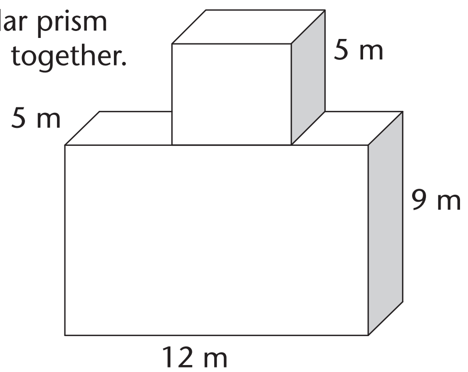
Volume of a triangular prism = $\frac{1}{2}(\text{base} \times h) \times \text{height of prism}$

Because we multiply three dimensions, the units used are cubic units, such as mm^3, cm^3 or m^3.

VOLUME CALCULATIONS

Calculate the volume of the following objects using the formulae given above.

1. A cube and rectangular prism are glued together.



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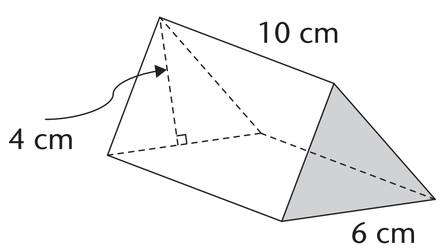
2.



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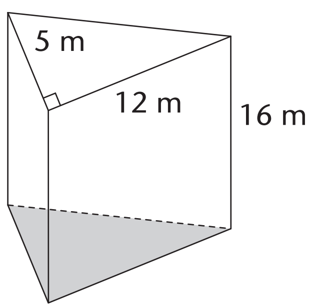
3.



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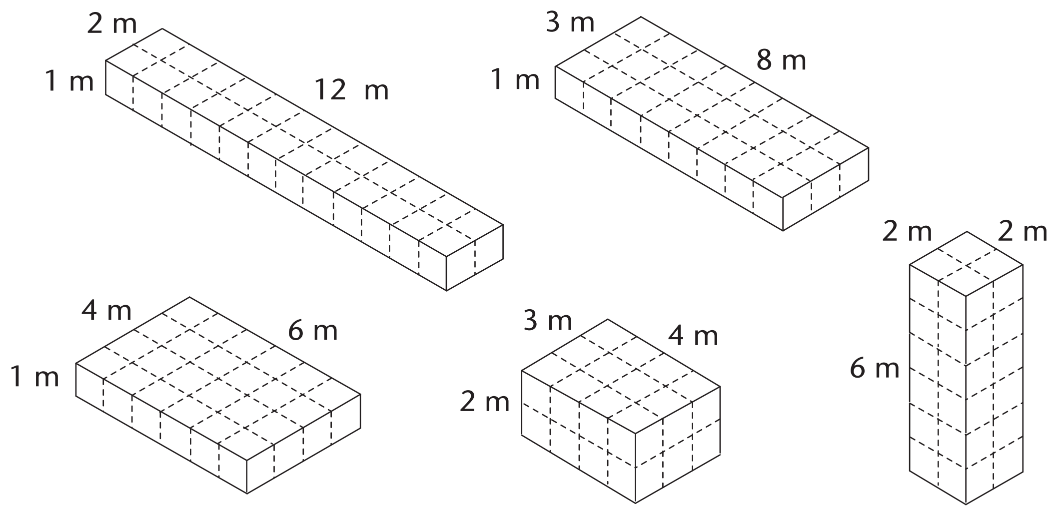
Maths2_Gr8_LB_Book.indb 80 2014/09/04 10:39:15 AM

5.4 Relationship between surface area and volume

Do objects with the same volume always have the same surface area? Do the investigation below in order to find out.

1. (a) Calculate the surface area and volume of the following five rectangular prisms by completing the table below.



Length (m)	Breadth (m)	Height (m)	Surface area (m^2)	Volume (m^3)
12	2	1		
8	3	1		
6	4	1		
4	3	2		
2	2	6		

(b) In the last row of the table, write another set of dimensions (*l*, *b* and *h*) that will give the same volume but a different surface area as the ones already recorded.

2. Look at the completed table. What can you conclude about the surface area and volume of objects?

....

....

3. A rectangular prism has a volume of 8 m^3. Write down two possible sets of dimensions. Draw the prisms below with their dimensions written on the drawings.

4. The following table shows surface area and volume calculations for cubes with different side lengths.

Side length of cube (m)	**Surface area (m^2)**	**Volume (m^3)**
1	6	1
2	24	8
3	54	27
5	150	125
8	384	512
10	600	1 000

(a) Look at the surface area column. Does the surface area increase or decrease as the side length of the cube increases?

(b) Look at the volume column. Does the volume increase or decrease as the side length of the cube increases?

(c) Does *volume* or *surface area* increase more rapidly when the side length of the cube increases?

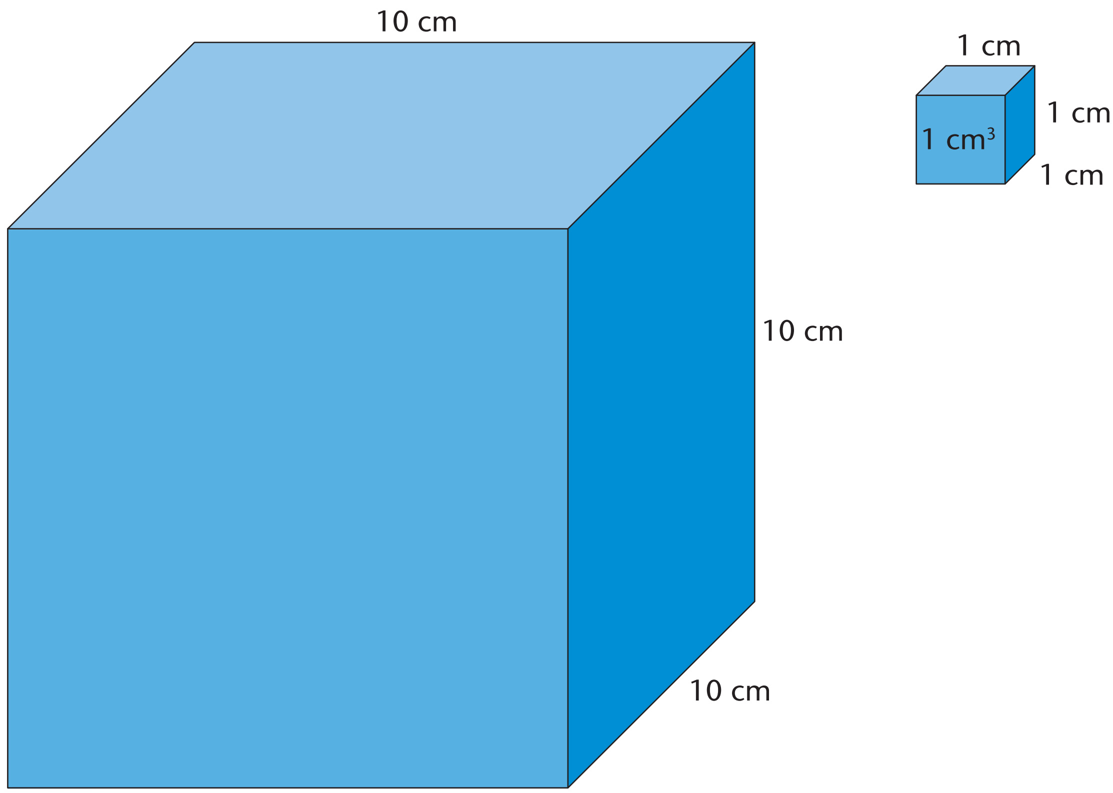
(d) Sketch a global graph of the volume of a cube versus its surface area.

5.5 Converting between cubic units

HOW MANY CUBES?

1. The small cube below has the dimensions 1 cm × 1 cm × 1 cm and a volume of 1 cm^3. How many 1 cm^3 cubes will you need to form a large cube with dimensions 10 cm × 10 cm × 10 cm like the one shown below?

....



2. How many 10 cm × 10 cm × 10 cm cubes will form a 100 cm × 100 cm × 100 cm cube?

....

3. (a) To form a 1 000 cm^3 cube you need 1 000 cubes with a volume of 1 cm^3. If cubes of 1 000 cm^3 (10 cm × 10 cm × 10 cm) are then used to form a cube of 100 cm × 100 cm × 100 cm, how many 1 000 cm^3 cubes will there be?

....

(b) What is the volume of this new cube? ....

(c) How many cubes of 1 cm^3 will form a cube with a volume of 1 000 000 cm^3?

....

4. Which of the cubes given below has a bigger volume? Explain.
   A. A cube with a volume of 1 m^3
   B. A cube with a volume of 1 000 000 cm^3

....

....

....

5. (a) How many 1 mm × 1 mm × 1 mm cubes (1 mm^3) are needed to form a 1 cm × 1 cm × 1 cm cube?

....

   (b) What is the total volume of the 1 mm^3 cubes forming the 1 cm^3 cube?

....

PRACTISE CONVERTING BETWEEN UNITS

When working with volume, you often have to convert between different cubic units. Here are two examples of how you can work out equivalent units.

Converting cm^3 to mm^3:

1 cm^3 = 1 cm × 1 cm × 1 cm
= 10 mm × 10 mm × 10 mm
= 1 000 mm^3

∴ multiply by 1 000

Converting cm^3 to m^3:

1 cm^3 = 1 cm × 1 cm × 1 cm
= 0,01 m × 0,01 m × 0,01 m
= 0,000001 m^3

∴ multiply by 0,000001 or divide by 1 000 000

1. Write the following volumes in cm^3.
   (a) 3 mm^3 ....
   (b) 45 mm^3 ....
   (c) 0,6 m^3 ....
   (d) 1,22 m^3 ....

2. Write the following volumes in mm^3.
   (a) 20 cm^3 ....
   (b) 151 cm^3 ....
   (c) 4,7 cm^3 ....
   (d) 89,5 cm^3 ....

3. Write the following volumes in m^3.

(a) 9 cm^3 (b) 50 cm^3

....

(c) 643 cm^3 (d) 1 967 cm^3

....

4. Write the following answers in cm^3.

(a) 4 m^3 + 68 cm^3 (b) 12 m^3 + 143 cm^3

....

....

5.6 Capacity of 3D objects

DIFFERENCE BETWEEN CAPACITY AND VOLUME

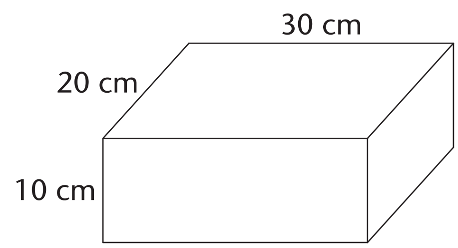
Capacity is the amount of space available *inside* an object.

Volume is the amount of space that the object itself takes up.

1. A solid block of wood measures 30 cm × 20 cm × 10 cm.

(a) What is its volume?

....



The same solid block of wood is carved out to make a hollow container. The measurements inside the container are 25 cm × 15 cm × 8 cm.

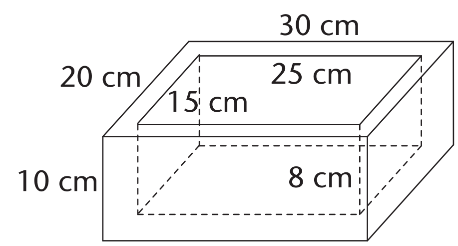
(b) How thick are the walls of the container?

....

(c) What is the capacity of the container?

....

(d) If you filled the container with water, what volume of water would the container hold?



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2. More of the wood is carved out of the container to make walls 1 cm thick at the sides and the bottom. Calculate the capacity of the container in litres.

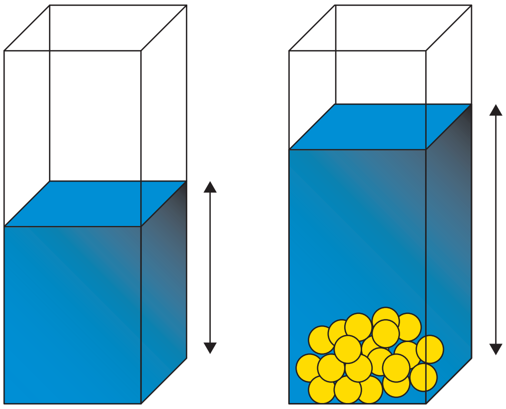
....

DISPLACEMENT AND MORE CAPACITY CALCULATIONS

Consider a glass vase half full of water. As soon as you place marbles into the water, the level of the water rises. This is not because the amount of water has changed, but rather because the marbles have taken the place of the water and have pushed the water higher up in the vase.

If one of the marbles has a volume of 1 cm^3, it would displace 1 ml of water.

∴ We know that:



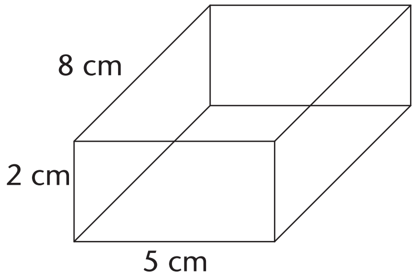
$1\ cm^3 = 1$ ml

$1\ m^3 = 1$ kl

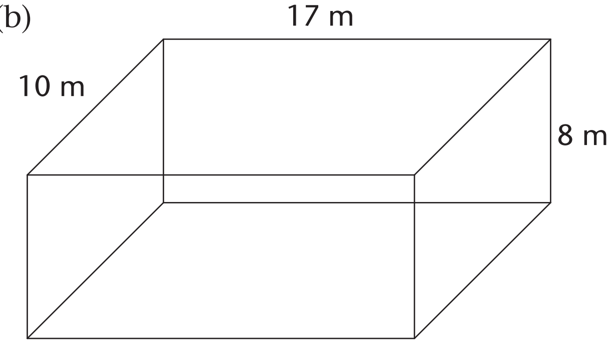
Displace means to move something out of its place.

1. Calculate the capacities of the following containers. The *inside* measurements are given. Write your answers in ml or kl.

(a)



(b)



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2. Work out a possible set of inside measurements for a container with a capacity of 12 kl. Draw a sketch and write the measurements on it.

Chapter 6
Collect, organise and summarise data

The data cycle is the process we follow when we do the following: pose a question, collect data to answer the question, organise and summarise the data sensibly, present the data in useful ways, interpret and analyse the data, and report on the data.

The activities in this chapter give you practice in collecting, organising and summarising data. Among other concepts, you will focus on: suggesting appropriate samples for an investigation; designing and using questionnaires with multiple-choice responses; organising data by using tally marks, tables, stem-and-leaf displays and grouped data; and summarising data by describing the mean, median, mode, range and extremes of the data set.

Nonkhanyiso	Saaliha	Herbert
Anna	Jennifer	Thabo
Mpho	Nomonde	Nomi
Nontobeko	Thandeka	Manare
Jonathan	Siza	Unathi
Sibongile	Prince	Gabriel
Dumisani	Duma	Hanna
Matsediso	Thandile	Simon
Chokocha	Nicholas	Miriam
Khanyisile	Jabulani	Sibusiso
Ramphamba	Nomhle	Mishack
Portia	Frederik	Peter
Erik	Lola	Maya
Jan	Adri	Thobele
Palesa	Jacob	Abraham
Kerishnie	Abdul	Sarita
Chris	Nina	Benjamin
Pieter	Doris	Cebisile
Jana	Ahmed	Zinzi
Duduzile	Gertruida	Nomcebo
Mohamed	Miemie	Tidimalo
Daniel	Erika	Otto
Qiniso	Zodwa	Ismael
Ofentse	Martinus	Andrew
Avhahumi	Muruwa	Sethunya

6 Collect, organise and summarise data

The term **data handling** is used to describe certain ways of trying to make sense of large collections of observations (data) about things in the real world. Data can be about many different things, for example people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

6.1 Collecting data

To find out more about any situation, we need to start by asking questions and collecting data. When you collect data, you need to consider:

- what you want to find out or the questions that you want to answer
- where you will find the data to answer the questions (for example from people such as learners in your school, your family and community; or from published sources such as newspapers, books or magazines)
- who you will collect the data from (all of the people or a sample)
- how you will collect the data (such as using questionnaires or interviews).

SOURCES OF DATA COLLECTION

In some cases you can use data that has already been collected by another person or organisation.

Example 1

Your question is:

What is the most common form of transport that learners in South Africa use to travel to school?

For this question, you will find that this data already exists in a publication called *Census @ School 2009*, published by Statistics South Africa. You can then present and interpret the existing data.

Example 2

Your question is:

What is the most common form of transport that learners at my school use to travel to school?

Check with the principal whether such data has already been collected by the school. If the data does not exist, or is very old, you need to decide where to get the data from. You could then decide to collect the data yourself from your peers.

For each of the following investigation questions, write down what or who would most likely be an appropriate source of information.

Question	Appropriate source of data collection
1. What is the favourite type of music amongst teenagers in my community?	
2. How much money do workers at Dress Factory earn per week?	
3. What is the height that baobab trees usually grow to?	
4. What are the ages of learners in Grades R to 7 in South Africa?	
5. How many people in South Africa have access to electricity?	
6. How many people in different African countries had malaria during the last five years?	
7. Has my school recycled more or fewer glass bottles this year than last year?	
8. What kind of household chores do 7- to 10-year-olds in my neighbourhood usually do?	
9. How many children in South Africa under the age of 10 have been vaccinated to protect them from childhood diseases?	

Maths2_Gr8_LB_Book.indb 90 2014/09/04 10:39:16 AM

POPULATIONS AND SAMPLES

The whole group of people (or things) that you want to find out about is normally called the **population**.

A population is often quite large. The size of the population depends on what you need to find out. The larger your population, the more difficult it becomes to ask each member of that population the questions you want to ask.

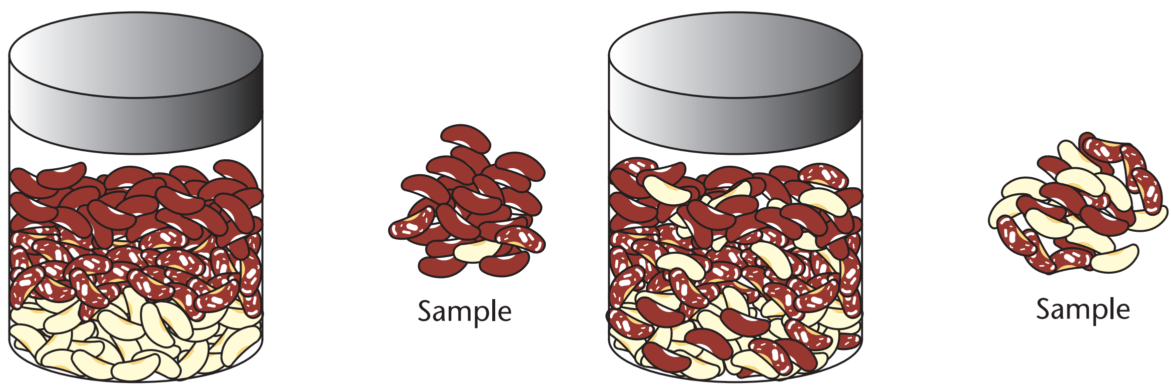
You could choose a smaller group of individuals from the population. Such a group, used to represent the whole population, is called a **sample**.

Examples

1. You may want to find out how much time the learners in a school spend on doing homework. If there are many learners in the school, you may be unable to ask them all about it. What you can do in this case is to talk to some learners in each class in the school. You may for instance speak to five learners from each class.
2. Health researchers may collect information about children by doing a survey of households selected randomly in each community.

RANDOM SAMPLES

A sample has to be chosen carefully to make sure that it represents the population. To understand what this means, think about what happens if you choose some beans from a jar in which the different kinds of beans are in separate layers. If you take a sample of the beans at the top, the sample is not representative. If the beans are all mixed up, then each bean has an equal chance of being chosen. The sample will be representative.



Example

Two ways you could select a random sample of learners from your school are:

1. You may write the names of all the learners on separate paper strips. You then put all the strips in a plastic bag, mix them, and draw 30 strips without looking at the names before you have finished.
2. You could select every fifth name from each of the class lists.

Look at the investigation questions. Which of the samples given do you think will reflect the whole population more appropriately? Tick your choice and give a reason.

Sample 1	**Sample 2**	**Reason**
1. What is the type of music liked by most teenagers in my community?		
50 teenagers at a local school	25 teenagers each from two different local schools	
2. How much money do the 200 workers at Dress Factory earn per week?		
The workers at every fourth workstation in the factory	The 50 workers that gather outside during lunch break	
3. What is the height that baobab trees usually grow to?		
All baobab trees in a marked-off area	Every second baobab tree in a marked-off area	
4. What are the ages of learners in Grades R to 7 in South Africa?		
All the Grade R to 7 learners in my school	Ten learners in each grade from Grade R to 7 at three different schools	
5. Has my school recycled more or fewer glass bottles this year than last year?		
All the glass bottles recycled in one month this year and in the same month last year	The glass bottles recycled in one month this year and in any other month last year	

QUESTIONNAIRES

We can use different methods to collect data, for example questionnaires, face-to-face interviews or telephonic interviews. In this section, you will work with questionnaire questions that have multiple-choice responses.

Here are two questions with multiple-choice responses from which a respondent will choose.

A **respondent** is a person who responds to the questions.

How satisfied are you with our level of service? ☐ Not satisfied at all ☐ Fairly satisfied ☐ Very satisfied	What is the colour of your eyes? ☐ brown ☐ green ☐ blue ☐ other

1. Write a suitable question with multiple-choice responses to find out the following information:
   (a) What do teenagers spend their money on?
   (b) How much time do Grade 8s spend on homework every day?

2. Choose one of the questions above. Write down what you think would be the best sample to use if you had to conduct this investigation.

   ....

   ....

   ....

3. Use the multiple-choice question that you chose in question 2 to collect the data. Keep the results for the next section.

6.2 Organising data

The way we organise and summarise data depends on the kind of data that we have. It also depends on what we want to find out from the data. Work in groups to explore this. Don't worry about getting the answers right at this stage. You will learn about the different ways to organise and summarise data in this chapter.

Look at the following sets of data. For each one, discuss with your group and write down what we want to find out and what you think we need to do to the data.

1. Data collected to find out which day would be best to have a soccer club practice:
   Twenty-five learners' preferred day for soccer practice

   Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Friday Friday Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

   ....

   ....

2. Data collected to find out whether 5-year-old children in a certain village have healthy body weights:
   Body weights of twenty-five children in kilograms, rounded off to the nearest 0,5 kg

   17 kg 16,5 kg 13,5 kg 14 kg 18 kg 18 kg 14 kg 21 kg 13,5 kg 15 kg 15 kg 14,5 kg 15,5 kg 19,5 kg 17 kg 17,5 kg 14 kg 14 kg 20 kg 14,5 kg 16 kg 18 kg 12 kg 16 kg 19 kg

   ....

   ....

   ....

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   ....

3. Data collected to find out how many learners answer a certain type of question in less than 20 seconds:
   Time taken (in seconds) by a group of learners to answer a question

   20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

   ....

   ....

   ....

   ....

4. Data collected to analyse the monthly salaries of employees of a small business:
The monthly salaries of ten employees

R8 000 R2 500 R75 000 R6 000 R7 500 R5 200 R4 800 R10 300 R15 000 R9 500

....

....

TALLY MARKS, TABLES AND STEM-AND-LEAF DISPLAYS

In Grade 7, you learnt about using tally tables and stem-and-leaf displays. We revise these two ways of organising data here.

We can use **tally tables** to record data in different categories. We draw a tally mark (|) for each item we count. We group tally marks in groups of five to count them quickly.

A **stem-and-leaf display** is a way of listing numerical data. If the numbers in a set of data consist of digits for tens and units (such as 23, 25, 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

Examples of tally marks:
A count of three = |||
A count of five = ~~||||~~
A count of seven = ~~||||~~ ||

If the numbers in a data set consist of three digits (such as 324, 428, 526), both the hundreds and tens digits are written in the stem column and the units digits are written in the leaf column, e.g. 32 | 4 would show 324.

Let's see how to display these numbers using the stem-and-leaf method:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56, 59

The numbers range from 12 to 59, so the first digits represent the numbers 10 to 50.

Key: 1 | 2 means 12

Stem	Leaves
1	2, 3
2	0
3	4, 5
4	7, 9
5	1, 3, 3, 3, 6, 9

Values with the same stem are written in the same row. Different leaves with the same stem are separated by a space or a comma.
In the first row, 1 | 2, 3 shows 12 and 13.

The stem column shows the tens digit of each value.

The leaf column shows the units digit of each value.

Notice that the stem-and-leaf display also shows us what the data set looks like. We can quickly see that most numbers are in the 50s and that there is only one number in the 20s.

1. Look back at the three sets of data on page 94. Fill in the table to show which form of data organisation you can use for each of these. Write a short explanation.

	Tally table	Stem-and-leaf display
A. Preferred day for soccer practice		
B. Body weight of children		
C. Time taken to answer a question		

2. Use the data set about preferred days for soccer practice:

Twenty-five learners' preferred day for soccer practice

Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Tuesday Friday Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

(a) Organise the data using a tally table.

Preferred day	Tally	Frequency

The **frequency** is the number of times that a day of the week appears in the list.

(b) Which day should they choose to have soccer practice? Why?

....

(c) Which day would be the worst day for soccer practice? Why?

....

3. Zandile collected data about the number of garments that each of her workers produced per day. The answers were as follows:

61, 58, 48, 59, 49, 51, 54, 67, 55, 70, 59, 60, 62, 59, 62, 63, 64, 48, 64, 55

(a) Record the data in the form of a stem-and-leaf display.

Key: ....

(b) Complete: Most values occur in the ....

(c) How many garments do the fastest and slowest workers make?

....

4. Use the data you collected in the investigation in question 3 on page 93.

(a) Decide whether a tally table or a stem-and-leaf display will organise the data best, and record the data in the space below.

(b) What does your tally table or stem-and-leaf display show about your data?

....

....

....

GROUPING DATA IN INTERVALS

When there are many values in a data set, it is often useful to group the data items into **class intervals**.

Example

Height in centimetres	Frequency
130–140	6
140–150	13
150–160	31
160–170	30
170–180	10

The class interval does not include the highest number in each case. So, the height of 150 cm falls into the interval 150–160 cm, not into the interval 140–150 cm.

This is a grouped frequency table. It represents 90 data values, but the values themselves are not shown. Instead, we show the frequency or the number of values falling into that interval.

1. The table shows the body weights (in kg) of athletes competing in a tournament.

55,2	56,1	58,4	59,3	60,6	61,2	61,7	63,4
63,2	64,2	65,9	66,5	66,7	67,3	67,8	68,0
70,5	72,9	73,4	74,1	74,8	75,9	76,7	78,7

(a) Group the weights into 5 kg intervals. List the intervals.

....

(b) Use a table to show the frequency of each class interval. It is useful to fill in the tallies first and then count up the frequencies, so that you don't leave any data items out.

Body weights of athletes	Tally	Frequency

(c) In which intervals are the highest numbers of athletes?

....

....

2. The following data shows the time taken (in minutes and seconds) by runners to complete a race.

34:30	34:59	35:36	36:58	40:08	40:55	41:33	43:18
44:26	45:40	48:13	48:49	49:15	50:08	52:09	53:36

(a) Group the times into suitable intervals. List the intervals.

....

(b) Record the grouped data in the form of a table.

(c) How long did the highest number of runners take to finish the race?

....

3. Take another look at the data about the time (in seconds) that learners took to answer a certain question:

20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

(a) Group the data into three intervals of 10 seconds. Fill in the table to show the grouped data.

(b) Do you think that learners will need at least 40 seconds to answer this type of question? Explain.

....

(c) Were there more learners who took at least 20 seconds or more to answer the question than learners who took less than 20 seconds? Explain.

....

....

6.3 Summarising data: measures of central tendency and dispersion

ONE NUMBER SPEAKS FOR MANY: THE MODE AND THE MEDIAN

1. A farmer wants to know whether he used good quality seed when he planted pumpkins. So he counts the number of pumpkins on each of a sample of 20 pumpkin plants. The numbers of pumpkins are given below.

   6 7 3 7 4 7 7 8 7 5 7 7 6 7 8 5 4 7 6 7

   (a) Arrange the data values from smallest to biggest, to get a clearer picture.

   ....

   (b) The farmer says to his wife: *Most of the plants have 7 pumpkins, so it is not too bad.* Do you think this is a good summary of the data, or should he say something more?

   ....

   ....

In some data sets some values or items are repeated often. The value or item that occurs most often is called the **mode**. Some data sets have more than one mode, and many data sets have none.

Instead of "most often" we can also say "most frequently".

   (c) Do you think that if the farmer had said the following, his wife would be somewhat better informed about the pumpkin plants?

   The number of pumpkins varies from 3 to 8, but there are 7 pumpkins on most of the plants.

   ....

2. Here are the Mathematics test results, out of 30, of a small class of 21 learners.

15	7	11	7	13	4	8	9	3	7	25
7	6	10	8	9	23	19	7	5	7	

   Bongile scored 9 out of 30 in the test, which is poor. Can he claim that his mark is in the top half of the class? Explain your answer well.

   ....

   ....

   ....

A set of data can be separated into a top half and a bottom half by arranging the data items from smallest to largest and finding the number between the two halves.

For example, the data set

23 35 44 21 28 32 38 41 39 42 24 27

can be rearranged like this:

21	23	24	27	28	32		35	38	39	41	42	44
bottom half							*top half*					

The number that sits halfway between the upper item in the bottom half and the lower item in the top half in this case is 33,5 (calculation: $[32 + 35] \div 2 = 33{,}5$).

The number that separates a set of data into an upper half and a lower half is called the **median**.

Half of the items are above the median and half of the items are below the median. To find the median, the data items need to be arranged from smallest to largest.

If a numerical data set has an odd number of items, the median is equal to the number in the middle of the set, when the items are arranged from smallest to largest:

3 4 5 6 7 7 7 7 7 7 **8** 8 9 9 10 11 13 15 19 23 25

3. Write down any eleven different numbers so that the median is 24.

....

4. The body weights in kg of 25 learners are given below, all rounded off to the nearest 0,5 kg. This data was collected to find out whether 5-year-old children in a certain village have healthy body weights.

17	16,5	13,5	14	18	18	14	21	13,5
15	15	14,5	15,5	19,5	17	17,5	14	
14	20	14,5	16	18	12	16	19	

Rearrange these data items into a bottom half and a top half, and state what the median body weight is.

....

....

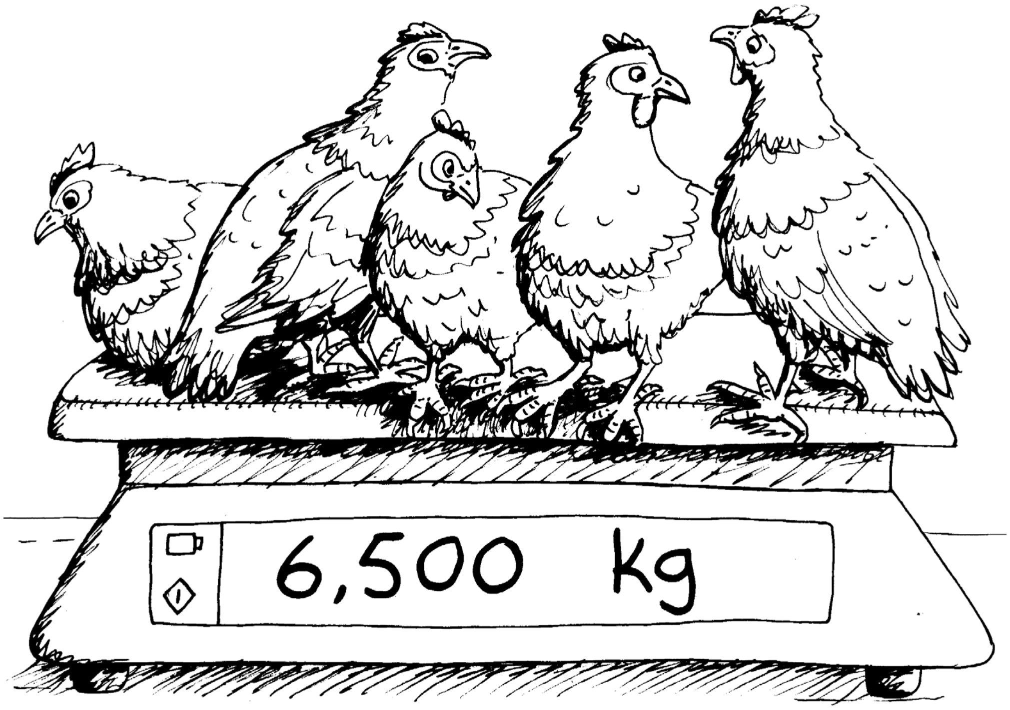
....

5. (a) Does the data set shown above question 3 have a mode, and if it has what is it?

....

(b) What is the mode of the data set in question 2 on page 96? ....

IF THEY WERE ALL EQUAL ... BUT THEY ARE NOT



1. Five chickens are put on a scale and the scale shows 6,500 kg which is the same as 6 500 g. What can you say if someone asks you:
   What does each of the chickens weigh?

   ....

   ....

   ....

2. A roadside stall owner has 10 watermelons to sell. They are not all of the same size and he did not pay the farmer the same price for each watermelon. So the stall owner weighs the watermelons and decides to sell them at the following prices:

   R16 R16 R18 R15 R14 R14 R16 R14 R13 R14

   (a) Check whether you agree that he will get R150 for all 10 watermelons together.

   ....

   (b) The stall owner now decides to make the prices of all the watermelons the same, to simplify advertising and selling. What should he make the price of each watermelon, if he still wants to receive R150 for all of them together?

   ....

3. Susan bought 6 pumpkins at the market. Her husband Abraham asks her what she paid for each pumpkin. Susan says:

   They actually came at different prices, and I have forgotten the prices now. But I know I paid R72 in total so it would have been the same if I paid R12 each. So you can say that on average I paid R12 each.

   (a) Check if Susan's answer to her husband is correct. The actual prices she paid for the different pumpkins are given below.

   R7 R15 R10 R16 R9 R15

   ....

   ....

   (b) How do you think Susan came to the R12 she used when she answered her husband's question?

   ....

   ....

   ....

   (c) Check if Susan's answer would have been correct if the actual prices of the pumpkins were as follows:

   R11 R12 R13 R11 R12 R13

   ....

When she gave an answer to her husband's question, Susan used the number 12 as a "summary" to represent the six different numbers 7; 15; 10; 16; 9 and 15. The number 12 is a good representation of 7; 15; 10; 16; 9 and 15 together because

$$\begin{aligned} & 7 + 15 + 10 + 16 + 9 + 15 \\ = \; & 12 + 12 + 12 + 12 + 12 + 12 \end{aligned}$$

If each value in a data set is replaced by the same number and the total remains the same, the "replacement number" is called the **mean** or **average.**

It can be calculated by dividing the total (sum) of the values by the number of values in the data set: Mean = sum of values ÷ number of values. (In the example above the mean is 72 ÷ 6 = 12.)

Like the median, the mean (average) may not be equal to any of the actual values in the data set.

4. Look again at question 1 on page 102 about the five chickens. If you want to now give a different answer than before, write it below.

....

....

....

5. A journalist investigated the price of white bread at different stores in two large cities. The prices in cents at 10 different shops in each city are given below.

City A:	927	885	937	889	861	904	899	888	839	880
City B:	890	872	908	910	942	924	900	872	933	948

(a) If you just look at the above data, do you think one can say that white bread is cheaper in the one city than in the other? Look carefully, and give reasons for your answer.

....

....

....

....

(b) Calculate the mean price of white bread for the sample in each of the two cities.

....

....

(c) Find the median bread price in the sample for each of the two cities.

....

....

....

....

6. Geoffrey is a stock famer. He buys 21 goats at a mean price of R830 each.

(a) How much do the 21 goats cost, in total?

....

(b) One of the goats was a stud goat for which Geoffrey paid R4 800.
What was the mean price of the other 20 goats?

....

....

7. (a) Find the mean and the median of this data set.

1 1 1 1 1 1 1 1 2 2 2 2 2 2 130

....

(b) Write ten numbers so that the mean is much smaller than the median.

....

(c) Write ten numbers so that the mean is much bigger than the median.

....

(d) Write ten different numbers so that the mean is equal to the median.

....

8. Here are the times taken by the different learners in Grade 8A in a certain school, in seconds, to do question 7(b) above.

20	30	36	14	20	14	29	39	15	37	35	24	
29	29	18	16	38	13	24	27	22	38	29	11	38

Here are the times taken by the different learners in Grade 8B in the same school, in seconds, to do question 7(b).

20	22	39	22	16	37	36	15	14	13	16	10	14
26	11	14	31	17	11	28	39	20	35	26	20	

Which class works the fastest, Grade 8A or Grade 8B? Explain your answer very well.

....

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HOW WIDE IS THE DATA SPREAD?

1. Two samples were taken from the eggs produced on two different egg farms, to investigate the masses of the eggs coming from the two farms.

   The mean mass of the eggs from farm A is 50,6 g and the median mass is 52,0 g.

   The mean mass of the eggs from farm B is 50,3 g and the median mass is 52,0 g.

   (a) Do these figures indicate that the eggs from the two farms are similar, or that they differ?

   ....

   (b) The actual masses of the eggs in the two samples are given below. Check whether the mean and median masses given above are correct.
   Masses of the sample of eggs from farm A, in grams:

   51 54 45 53 49 54 55 46 54 45

   Masses of the sample of eggs from farm B, in grams:

   53 52 55 44 57 41 59 43 47 52

   ....

   ....

   ....

   ....

   (c) In what way do the egg masses from the two farms differ?

   ....

   ....

The **range** of a set of data is the **difference** between the **maximum** (highest or top value) and the **minimum** (lowest or bottom value).

The values in the data set below vary from 36 to 60, hence the range is 60 – 36 = 24.

36 36 39 39 43 45 46 47 52 52 53 55 57 60

2. The following data shows the exam marks of two groups of learners.

   Group 1: 30 31 35 50 55 58 60 70 78 80 88 88 90 90
   Group 2: 55 55 56 57 59 59 59 67 69 75 80 80 80 81

   Compare the two groups by completing the following statements.

   (a) In group 1 the marks vary from .... to ...., a range of ....

   (b) In group 2 the marks vary from .... to ...., a range of ....

3. These two sets of data show the prices of houses that have been sold in towns A and B in one month:

Town A:	R321 000	R199 000	R181 000	R303 000
	R299 000	R248 000	R283 000	R315 000
	R405 000	R380 000	R322 000	

Town B:	R88 000	R122 000	R175 000	R166 000
	R107 000	R105 000	R1 114 000	R100 000
	R151 000	R1 199 000	R146 000	

(a) Read through the prices in each list and write down anything that comes to your mind when you look at the two sets of figures.

....

....

(b) You have been asked to write a short paragraph on the house prices in the two towns, for the local newspaper. You want to make it quick and easy for the readers to get some sense of the house prices in the two towns. Work in the space below and then write your newspaper paragraph neatly in the frame.

The mean price for houses in the list for town A on the previous page is R296 000. This is very close to the median of R303 000. All the prices in town A are within R115 000 of the mean.

The mean price for houses in the list for town B is R315 727, which is more than double the median price of R146 000. Nine of the eleven houses in town B cost far less than the mean, while the prices in town A are more evenly spread on both sides of the mean.

4. Someone asks you about the house prices in towns A and B, and you say: *The mean house price in town A is R296 000, and the mean house price in town B is R315 727.*
   (a) Does this statement provide good information about the difference in the house prices in the two towns? In what way may it actually be misleading?

   ....

   ....

   ....

   (b) What causes the mean to be a misleading way of describing the data for the house prices in town B?

   ....

Data items like the house prices of R1 114 000 and R1 199 000 in the list for town B in question 3 are called **outliers** (or extreme values). Outliers are data values that are much lower or much higher than any other values in the data set. The mean is not a good way to summarise a set of data with outliers.

5. (a) Is there an outlier in this set of monthly salaries of the employees at a small business?

R8 000	R2 500	R75 000	R6 000	R7 500
R5 200	R4 800	R10 300	R15 000	R9 500

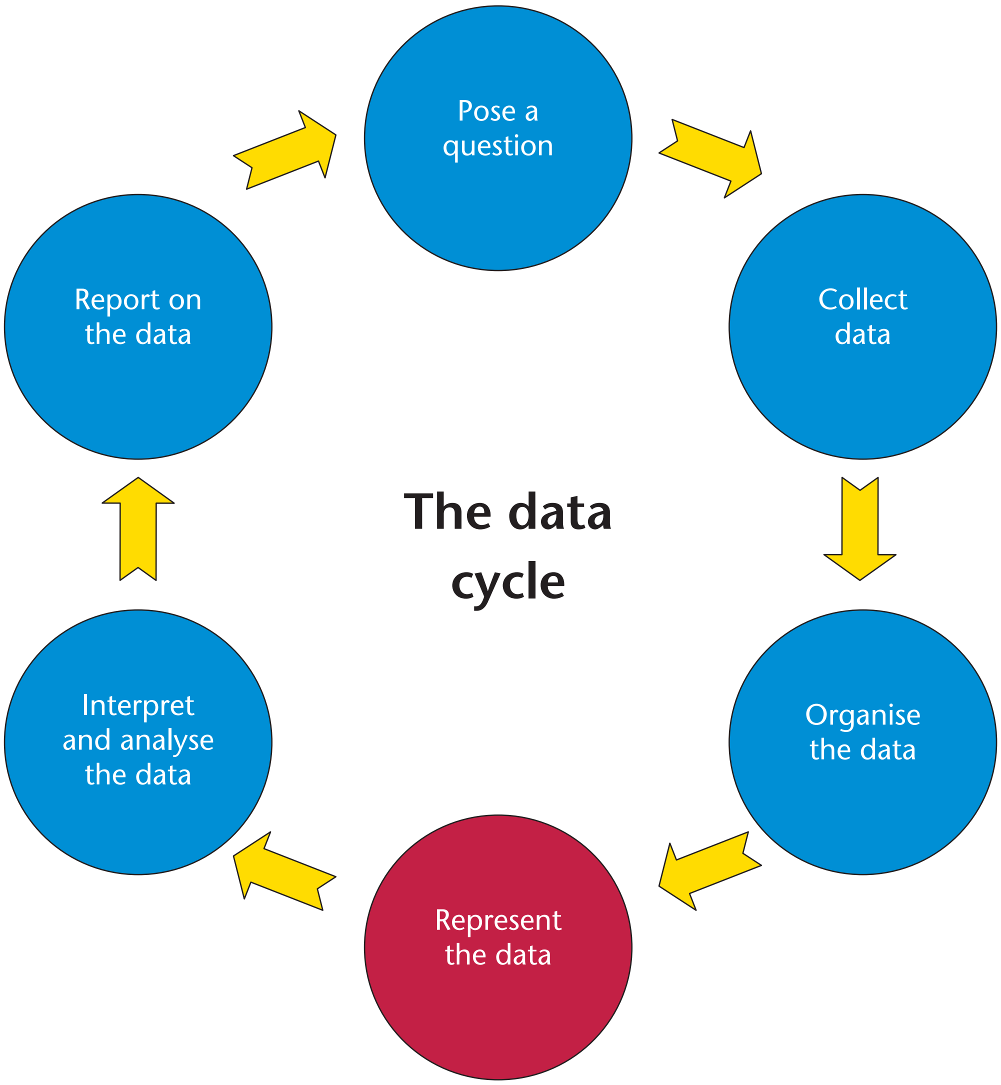
   ....

   (b) Would the median prices of the two data sets be a good way to indicate the main difference between house prices in towns A and B in question 3? Explain your answer.

   ....

Chapter 7
Represent data

In the previous chapter, we focused on collecting, organising and summarising data. Now we focus on representing data in bar graphs, double bar graphs, histograms, pie charts and broken-line graphs. You have already learnt how to represent data in all these forms, except for broken-line graphs, in previous grades.

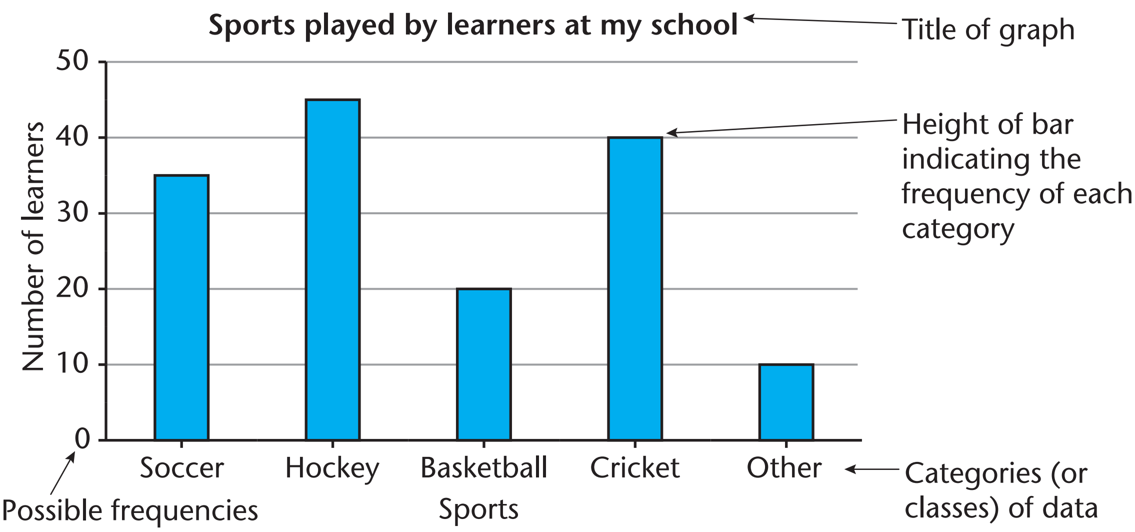

Pose a question
Collect data
Organise the data
Represent the data
Interpret and analyse the data
Report on the data
The data cycle

7 Represent data

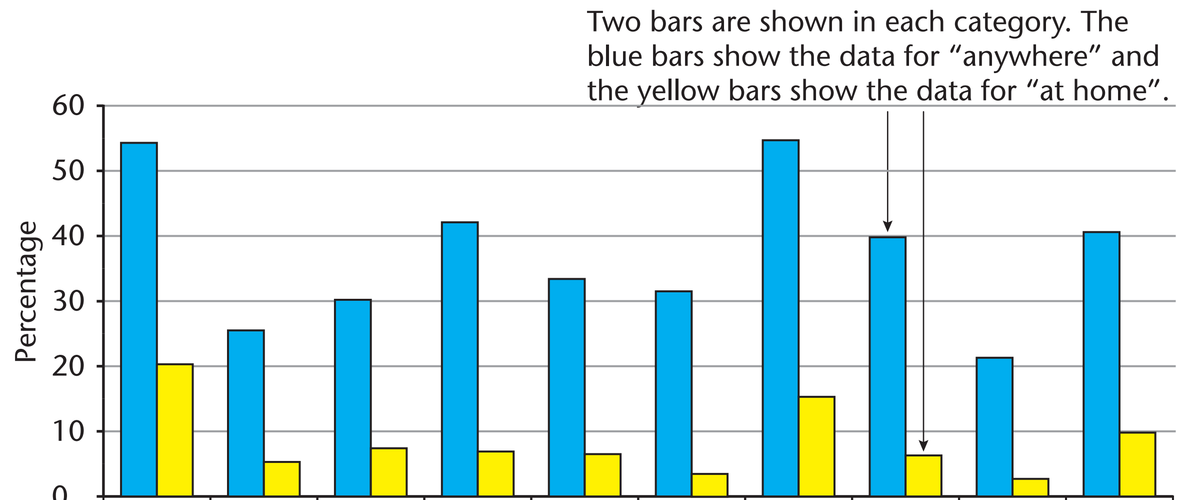
7.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

A **bar graph** usually shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis, for example:



A **double bar graph** shows two sets of data in the same categories on the same set of axes. The graph below shows the percentage of households *with* access to the internet at home, or for which at least one member has access elsewhere, by province in 2012.



	Western Cape	Eastern Cape	Northern Cape	Free State	KwaZulu-Natal	North West	Gauteng	Mpuma-langa	Limpopo	South Africa
Anywhere	54,3	25,5	30,2	42,1	33,4	31,5	54,7	39,8	21,3	40,6
At home	20,3	5,3	7,4	6,9	6,5	3,5	15,3	6,3	2,7	9,8

A key explains the colours used to distinguish the two sets of data.

REPRESENTING DATA IN BAR GRAPHS AND DOUBLE BAR GRAPHS

1. Road accidents are a big problem in South Africa, especially during the holiday season. Statistics about road accidents are published to make people aware of this problem.

   (a) Round off the numbers in the second column to the nearest hundred and write the result in the third column.

Year	Number of road accident deaths	Rounded-off number
2002	3 661	
2003	4 445	
2004	5 234	
2005	5 443	
2006	5 639	

   (b) Draw a bar graph of the rounded-off numbers.

(c) What trend do you notice in this data?

....

(d) For this form of representation, do you think it makes a difference that you have rounded the data off? Explain.

....

....

2. Road accident data can be analysed in different ways. The table below shows the kinds of vehicles and the number of accidents that they were involved in for 2011. The data is from the Arrive Alive campaign.

Vehicle type	Number of accidents	Rounded-off number
Cars	6 381	
Minibuses	1 737	
Buses	406	
Motorcycles	289	
LDVs and Bakkies	2 934	
Trucks	861	
Other and unknown	1 161	
Total	**13 769**	

(a) A large proportion of the data involves "other and unknown" vehicle types. What could the reason be for this?

....

(b) What information is missing from the table? What would we need to know to get a better picture of these accidents?

....

....

(c) Which vehicle was involved in the highest number of accidents? Does this mean that this vehicle is the least safe? Explain.

....

....

....

....

(d) Round off the data in the table on the previous page to the nearest 100. Then draw a bar graph of the rounded-off data.

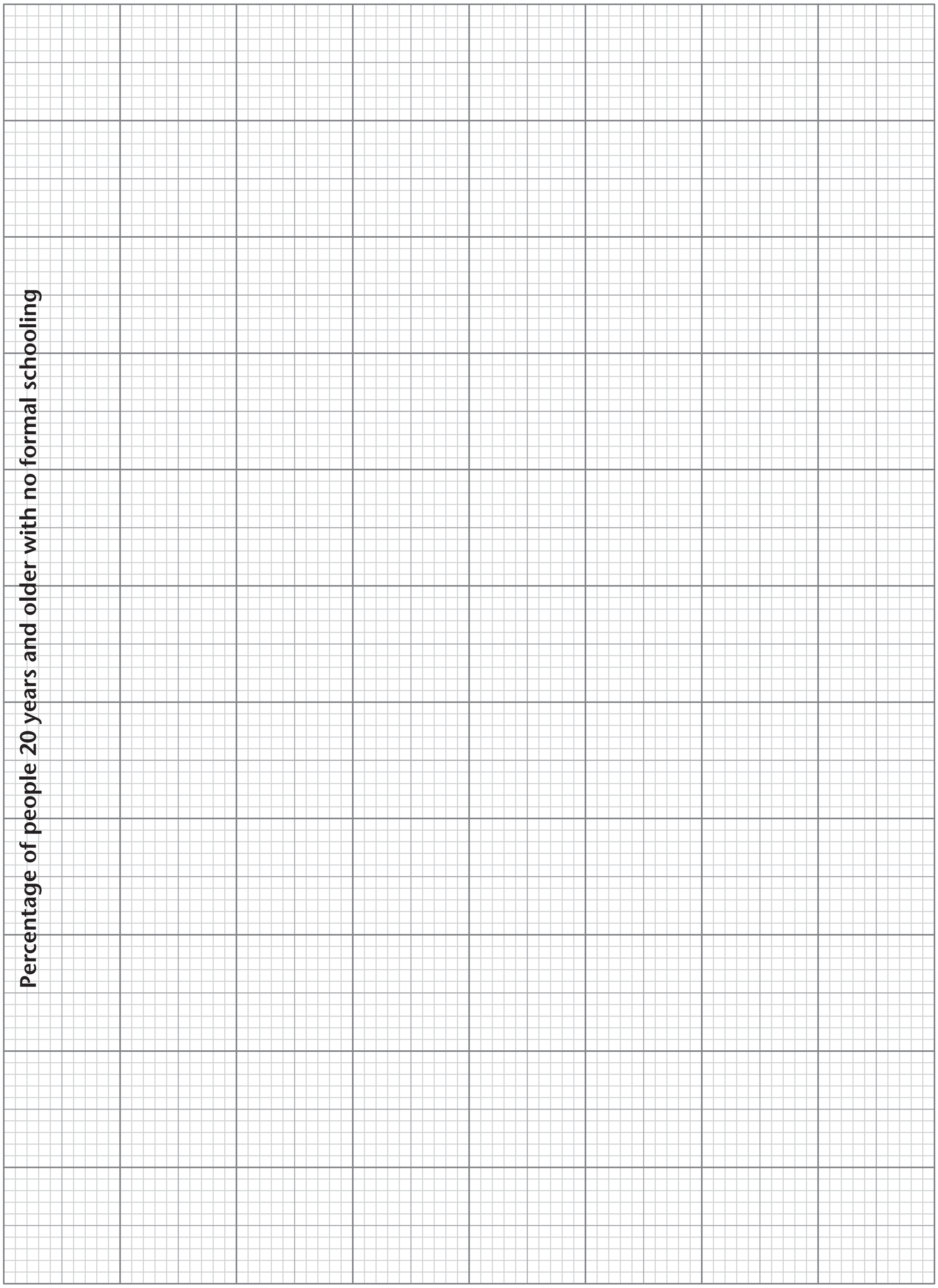
3. Statistics South Africa released the data below in their 2012 General Household Survey.

Percentage of people 20 years and older with no formal schooling

	WC	EC	NC	FS	KZN	NW	GAU	MPU	LIM
2002	4,4	12,5	16,5	10,0	11,8	14,6	4,5	17,1	20,1
2012	1,5	6,4	8,5	4,8	7,8	8,8	1,9	10,6	11,6

(a) Why do you think the data from 2012 is compared to 2002?

(b) Plot a double bar graph of this data on the next page.


Percentage of people 20 years and older with no formal schooling

(c) Explain the data for Limpopo, by filling in the missing percentages below: The percentage of people over 20 who had no formal schooling in Limpopo in 2002 was .... In 2012 the survey showed that the percentage of people with no formal schooling was .... The difference in the percentages is ....

(d) From the graph, which provinces showed the least change in the percentage of people with no formal schooling? Explain how you know this and give a suggestion about why this could be so.

....

....

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....

....

7.2 Histograms

WHAT HISTOGRAMS REPRESENT

A histogram is a graph of the frequencies of data in different **class intervals**, as shown in the example below. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

A histogram looks somewhat like a bar graph, but histograms are normally drawn without gaps between the bars.

Example

The numbers of oranges harvested from 60 trees in an orchard are given below.

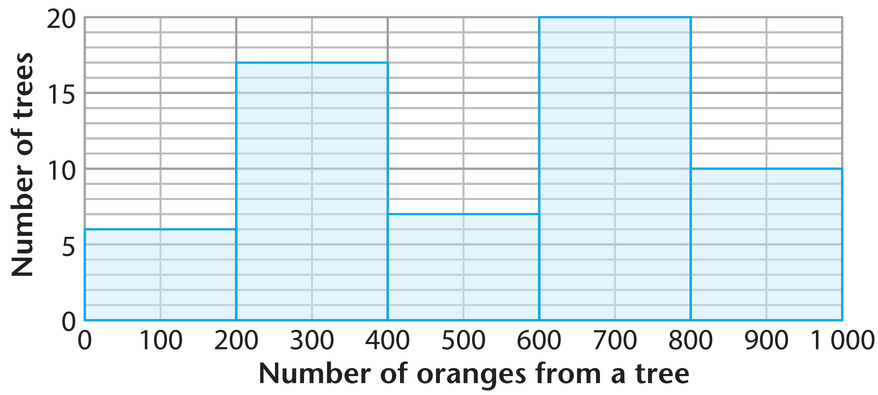
830	102	57	726	400	710	333	361	295	674	927	945
276	792	787	765	540	785	305	104	88	203	224	974
852	716	790	145	755	661	728	637	319	221	766	764
397	734	856	775	330	659	211	918	345	360	518	822
818	727	346	279	804	478	626	324	478	471	69	462

The frequencies of trees with numbers of oranges in specific class intervals are shown in this table.

Number of oranges	Number of trees
0–200	6
200–400	17
400–600	7
600–800	20
800–1 000	10

We follow the convention that the top value (also called the upper boundary) of each class interval is not included in the interval. The value of 400 is therefore included in the interval 400–600 and not in the interval 200–400.

Here is a histogram of the above data.



REPRESENTING DATA IN HISTOGRAMS

1. In the 2009 Census@School, the learners from Grades 3 to 7 at a certain school were asked how long (in minutes) it takes them to travel to school. The table shows the results from a sample of 120 learners.

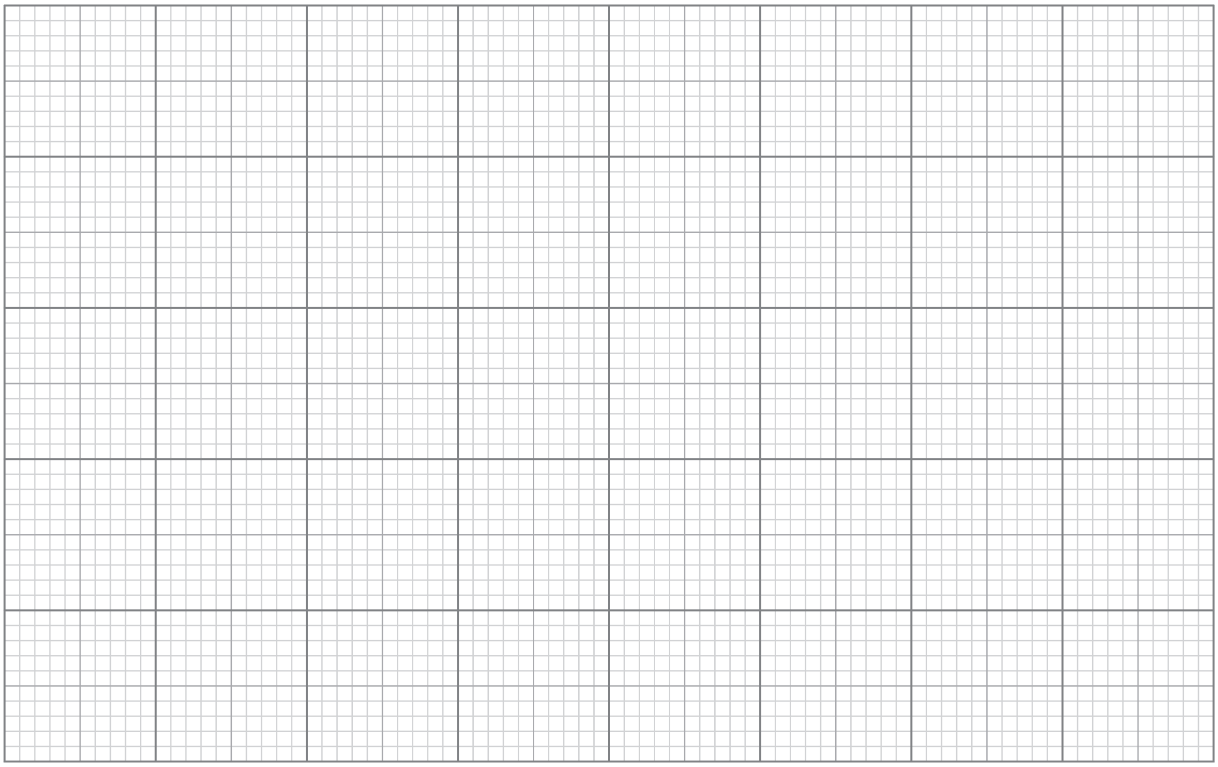
Time in minutes	Frequency
0–10	15
10–20	48
20–30	34
30–40	14
40–50	6
50–60	1
60–70	2

(a) In which interval is a travel time of 30 minutes counted?

....

....

(b) Draw a histogram to illustrate this data.



(c) Describe in your own words what the histogram shows.

....

....

....

....

(d) What would you expect this data to look like for a school in a farming area?

....

....

2. Company A manufactures light bulbs. They want to see how many hours (h) their light bulbs last, as they would like to use that data to promote their light bulbs. They investigate a sample of 200 light bulbs straight from the factory. This is the data they collect.

Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	15	25	70	50	40

(a) Draw a histogram of this data.

(b) Company B, which makes similar light bulbs, carries out a similar experiment and gets the following results. Draw a histogram of the data.

Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	7	11	24	18	0

(c) Comment on the differences between the two histograms.

....

....

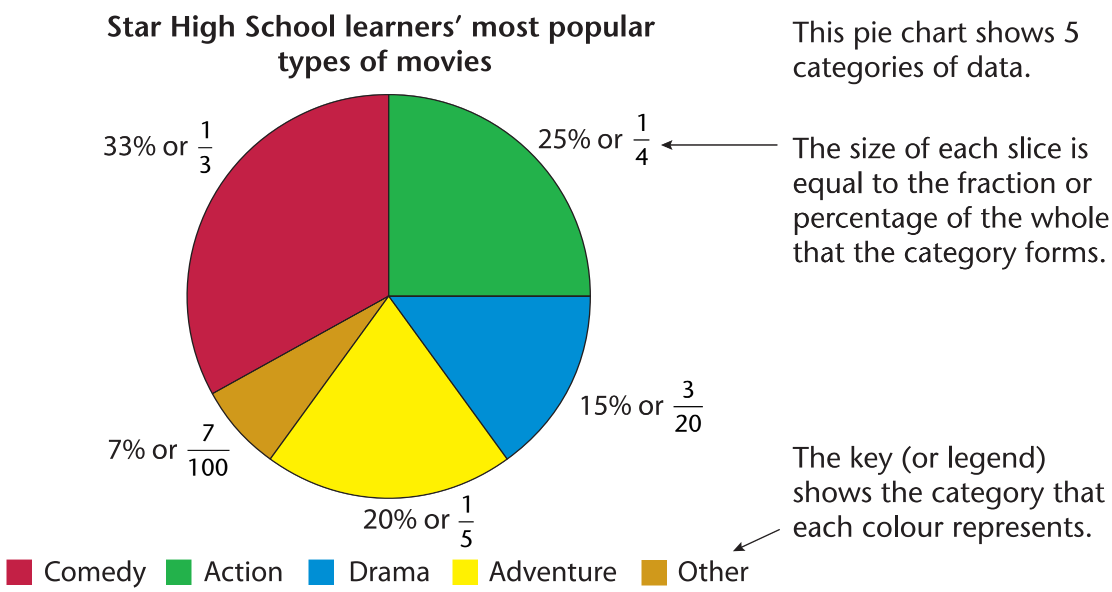
....

....

....

7.3 Pie charts

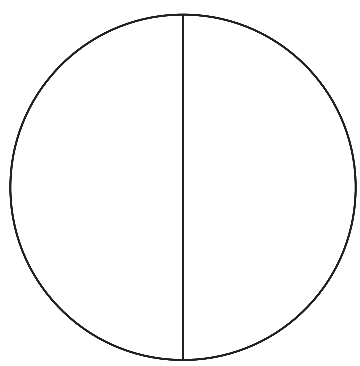
A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle. The whole graph shows how much each category contributes to the whole.



ESTIMATING THE SIZE OF SLICES IN A PIE CHART

In Grade 7, you learnt how to estimate the fractions or percentages of a circle in order to draw pie charts.

1. (a) Complete the following pie chart to show that $\frac{1}{2}$ of the class walk to school, $\frac{1}{4}$ travel by train and $\frac{1}{4}$ travel by car.



(b) Determine what percentage of learners:
walk ....
travel by train ....
travel by car ....

(c) There are 40 learners in the class. Determine how many:
walk ....
travel by train ....
travel by car ....

2. The following data shows the highest level of schooling completed by a group of people.

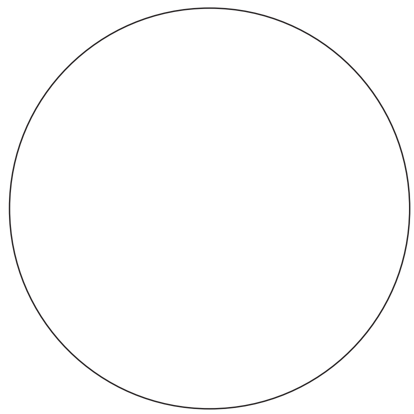
Highest level of schooling completed	Number of people	Fraction of whole	Percentage of whole
Some primary school grades	36		
All primary school grades	54		
Some high school grades	72		
All high school grades	18		
Total	**180**		

(a) How many people make up the whole group? ....

(b) Complete the third column by working out the fraction of the whole group that each category makes up.

(c) Complete the fourth column by working out the percentage of the whole group that each category makes up.

(d) Draw a pie chart on the next page, showing the data in the completed table. (Estimate the size of the slices.)



7.4 Broken-line graphs

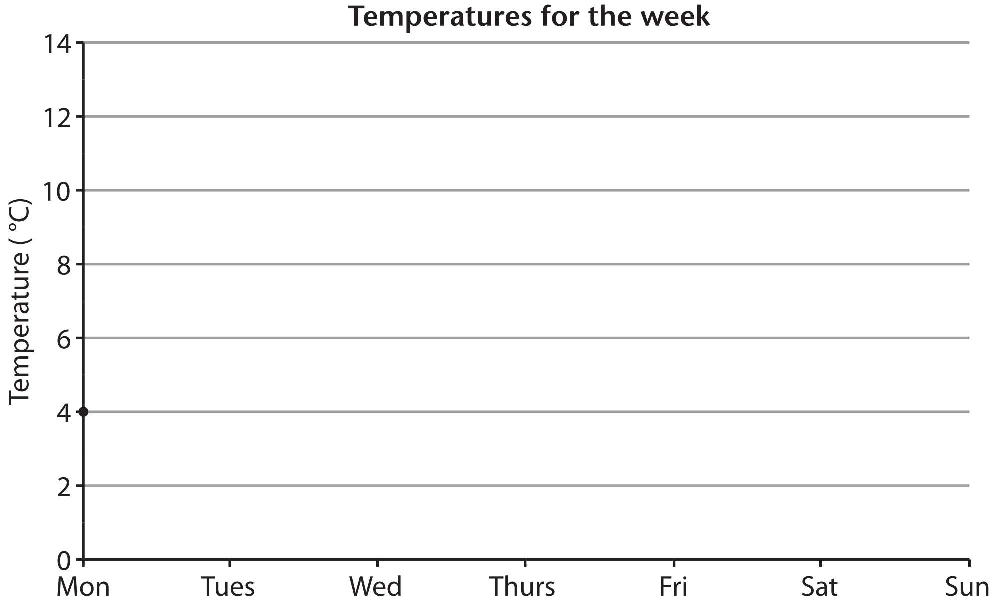
PLOTTING DATA POINTS

The table shows the average temperature in Bethal recorded every day for one week.

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Temperature (°C)	4	10	12	9	13	13	11

1. Plot the data on the set of axes below. Make a dot for every point that you plot.

Temperatures for the week



2. Use a ruler to join the dots in order.

You have drawn a broken-line graph.

> A **broken-line graph** is a line that joins consecutive data points plotted on a set of axes. Broken-line graphs are useful to show how something has changed or stayed the same over time.

DRAWING BROKEN-LINE GRAPHS

The table shows the income of Pam's small business and Luthando's small business over 6 months.

Month	January	February	March	April	May	June
Pam's income (R)	12 000	12 000	9 000	6 000	7 000	9 000
Luthando's income (R)	6 000	7 000	8 000	8 000	9 000	9 000

1. Draw a broken-line graph showing Pam's income.

Maths2_Gr8_LB_Book.indb 123 2014/09/04 10:39:21 AM

2. Draw a broken-line graph showing Luthando's income.

3. Whose income seems to be increasing steadily per month? ....

COMPARING DIFFERENT WAYS OF REPRESENTING DATA

The table on the next page shows data from the 2012 General Household Survey (Statistics South Africa).

1. Is it possible to find the mean, median and mode of this data? Explain.

....

....

....

....

Mode of school transport for learners in numbers and percentages

Mode of transport	Statistic (numbers in thousands)	Usual transport to school
Walking	Number	10 549
	Percentage	68,9
Bicycle/motorcycle	Number	90
	Percentage	0,6
Minibus taxi/sedan taxi/bakkie taxi	Number	1 129
	Percentage	7,4
Bus	Number	434
	Percentage	2,8
Train	Number	94
	Percentage	0,6
Minibus/bus provided by institution/government and not paid for	Number	209
	Percentage	1,4
Minibus/bus provided and paid for by the institution	Number	88
	Percentage	0,6
Vehicle hired by a group of parents	Number	1 344
	Percentage	8,8
Own car or other private vehicle	Number	1 371
	Percentage	8,9
Subtotal	Number	15 308
	Percentage	100

2. What are two good graphs you could use to represent this data? Explain your answer.

....

....

....

....

....

....

3. Describe the advantages of each of these ways (the two graphs and the table) for this particular set of data.

....

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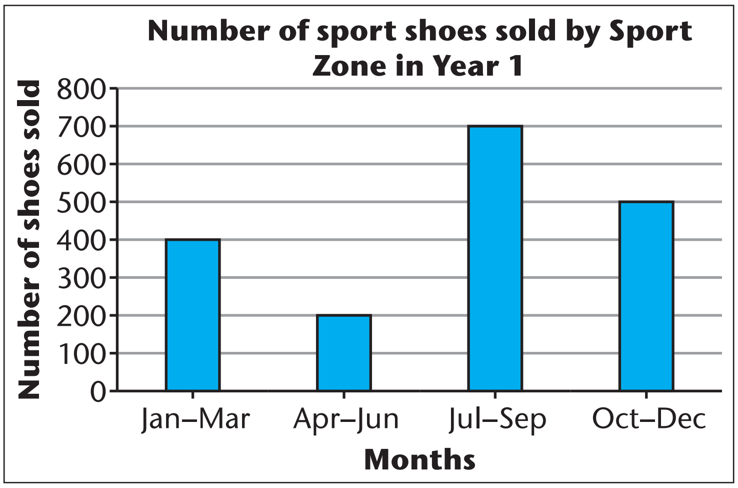
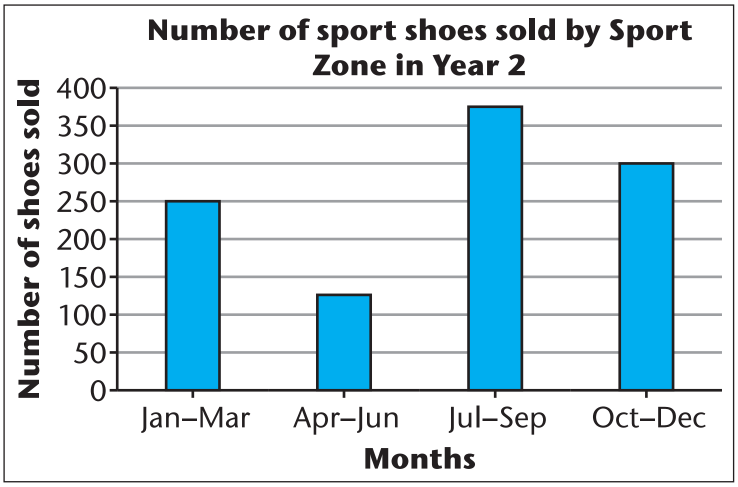
....

4. Draw the two graphs that you named in question 2 in your exercise book.

CHAPTER 8
Interpret, analyse and report on data

In this chapter, you will develop and practise some critical data analysis skills. This means looking at reported data and analysing the whole data handling cycle for this data. You need to decide which way of representing data is best in a given situation and identify data that is hidden rather than shown. In summarising data, some measures are more appropriate for different types of data and for indicating central tendencies in the data. You also need to recognise some ways in which bias can appear in data, including methods of collecting, representing and summarising data.


Number of sport shoes sold by Sport Zone in Year 1
Number of shoes sold
800
700
600
500
400
300
200
100
0
Jan–Mar
Apr–Jun
Jul–Sep
Oct–Dec
Months


Number of sport shoes sold by Sport Zone in Year 2
Number of shoes sold
400
350
300
250
200
150
100
50
0
Jan–Mar
Apr–Jun
Jul–Sep
Oct–Dec
Months

8 Interpret, analyse and report on data

8.1 Critically analysing how data is collected

Data collection methods can sometimes result in bias and misleading data. This is not always intended by the researcher – it often happens when the source of the data was not carefully checked or the method of collecting data has not been planned carefully.

In chapter 6 you learnt that a sample must be large enough to be representative and must be randomly selected from the population. If data is collected from only one part of a population, it could be biased towards that part. The researcher has to be aware of all the places where bias could occur, and should design the data handling process so that it does not happen.

When you read reported statistics, always be aware that you need information about how the data was collected, when it was collected and how the sample was chosen.

Data can change over time, so you should also be aware of when it was collected. This information should be given in any report on data.

DATA SOURCES AND COLLECTION METHODS

1. Read the following paragraph and answer the questions that follow.

   A recent study revealed that 50% of high school learners smoke cigarettes, 45% drink alcohol and 60% abuse drugs. This is an indication of the general poor health and social problems of the teenage population in our country.

   (a) Do you agree that the figures are high enough to conclude that the habits of these teenagers are unhealthy?

   ....

   (b) Can we tell the following from the data above?

   - What the sample of this study was ....
   - Where the data was collected ....
   - When the data was collected ....

   (c) If the sample consisted of 10 teenagers all located in an area known for drug and alcohol abuse, would the data be a good reflection of all teenagers in the country?

   ....

(d) Describe what you think would be a better sample.

....

....

....

(e) Why is it important to know the date when this data set was collected?

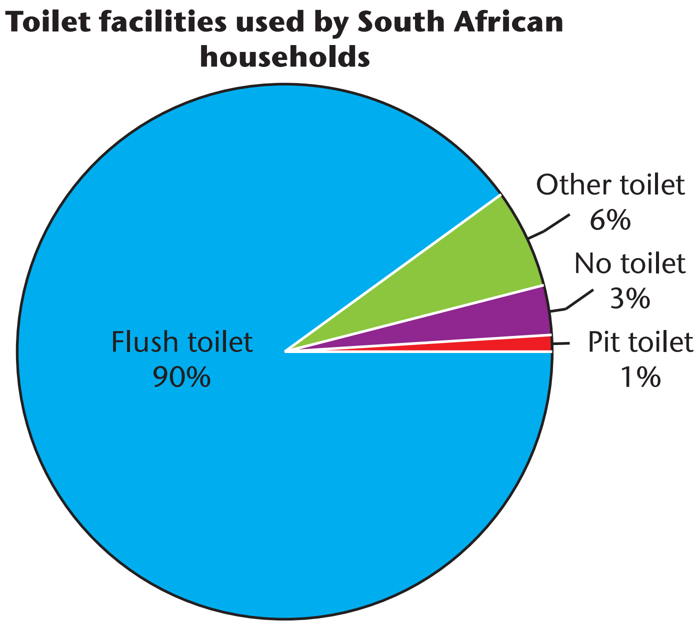
....

....

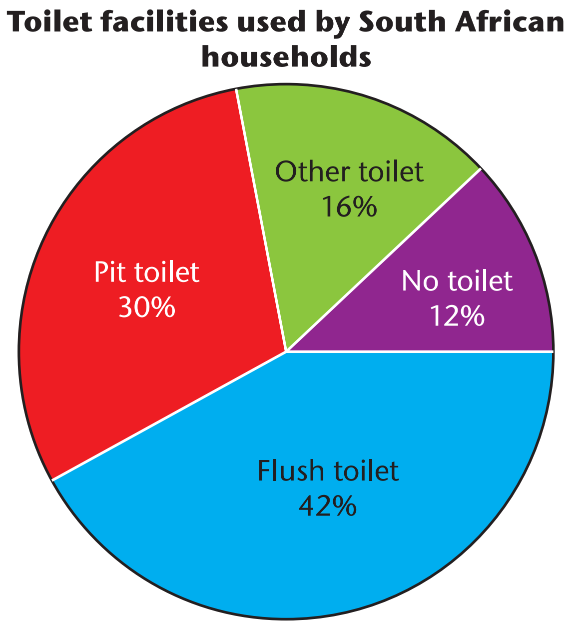
....

....

2. The following pie charts show the toilet facilities in households in South Africa.



Pie chart A



Pie chart B

(a) According to pie chart A, what type of toilet facility do most people have and what percentage of households is this?

....

(b) How will your answer for (a) be different if you use pie chart B to answer the question?

....

(c) Write a short report in one paragraph about the data in the pie charts.

....

....

....

....

(d) What can you conclude from the data in pie chart A?

....

(e) What can you conclude from the data in pie chart B?

....

....

(f) How could the year of data collection account for the differences in the data?

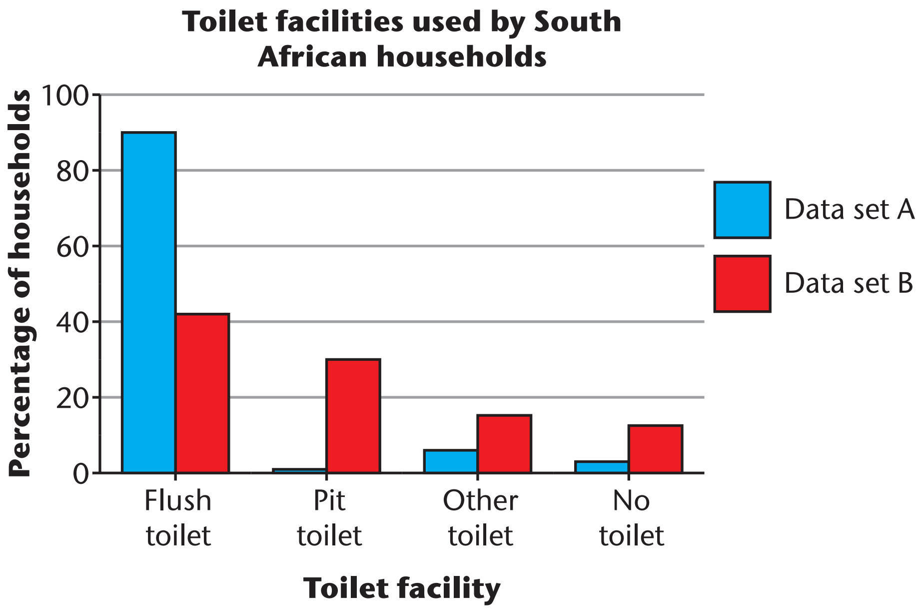
....

....

....

(g) The graph below shows the same data as the two pie charts. Do the pie charts or the double bar graph allow us to compare the two sets of data more easily?

....



(h) Do the pie charts or the double bar graph show the percentage of types of toilet facilities used in South Africa better?

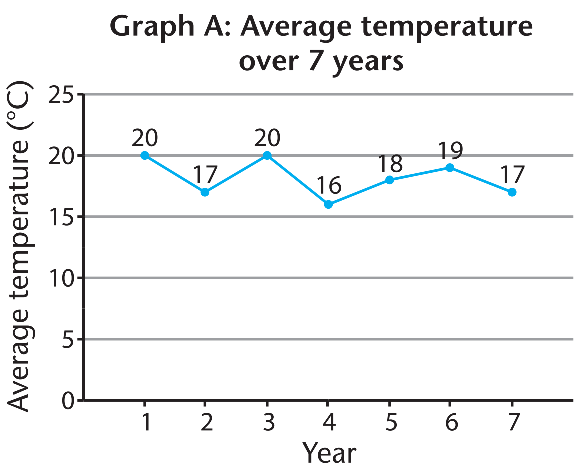
....

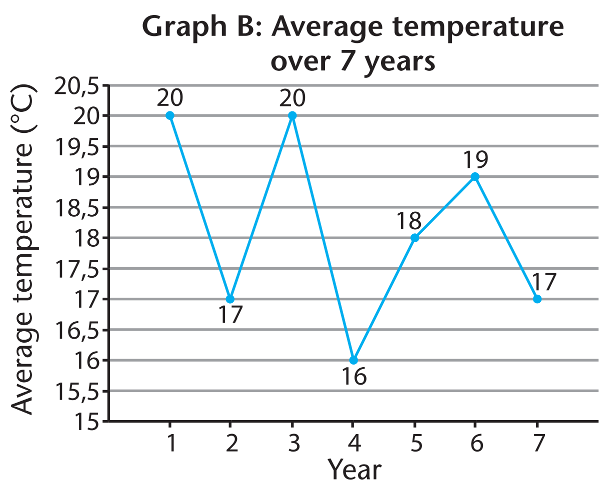
8.2 Critically analysing how data is represented

Graphs don't always show what they seem to show at first glance! If you look a bit closer, you might see that they are misleading you into drawing an incorrect conclusion. Work through the activity below to see how this can happen.

MANIPULATION IN DATA PRESENTATION

The graphs show the average temperatures recorded in the same place, at the same time.





1. Do both graphs show exactly the same data? ....

2. Why do the graphs look so different?

....

....

....

....

3. Which of the graphs would people use to emphasise that there are big differences in the temperatures over the years? Explain your answer.

....

....

....

....

4. Suggest a way to change the vertical scale of Graph A to emphasise even more that there are no big differences between the temperatures over the years.

....

5. Write a short report on Graph A. Also include a prediction of the temperatures in Years 8 and 9.

....

....

....

8.3 Critically analysing summary statistics

It is sometimes necessary to inform another person about a set of data that you have worked on. In doing that, you may want to save the other person having to look at all the values in the data set. You also want to emphasise some aspects of the data. It is for these purposes that we use summary statistics like the following:

- measures of central tendency (typical values): **mode**, **median** and **mean**
- measures of dispersion (values indicating the spread of data): the smallest and largest values and the difference between them (the **range**)

Summary statistics do not provide full information on data. Some information is always lost and so summary statistics can be misleading, especially if there are **outliers**, in other words values that differ a lot from the majority of the values.

HOW SUMMARY STATISTICS CAN BE MISLEADING

1. The manager of a small business is asked what monthly salaries his employees get. His answer: *The mean of the salaries is R13 731.*
   (a) Do you think the manager's answer is a good description of the salaries?

   ....

   ....

   ....

   (b) In order to have some sense of the salaries paid at the firm, which one of the following would you prefer to know: the *median* or the *mode* or the *range* or the *lowest and highest* salaries?

   ....

2. The actual monthly salaries of the 13 staff members in the small business mentioned in question 1 are given below.

R3 500	R3 500	R3 500	R3 500	R3 500
R4 200	R4 200	R4 200	R4 400	R12 000
R28 000	R44 000	R60 000		

In what ways may you be misinformed if you do not know the above figures, but only know that the mean salary is R13 731?

....

....

....

....

3. If only one summary statistic is used to provide information about the salaries at the firm, which of the following do you think would be the best to give? Give reasons for your choice.
   A. The mode
   B. The range
   C. The median
   D. The lowest and highest salaries

....

....

....

....

4. The different monthly salaries of employees at another small business are given below.

R34 000	R35 000	R3 400	R31 000	R32 000

(a) Why would the mean not be a good way to summarise this data?

....

....

....

(b) Calculate the mean salary.

....

....

5. The following data shows the number of boxes of chocolates sold by a store in 10 consecutive months.

   42 38 179 40 43 40 48 39 41 42

   (a) Which would be the better summary description of the data, the mean or the median? Explain your answer.

   ....

   (b) Write a good summary description of the data without using the median.

   ....

   ....

   (c) Would it make sense to leave out the outlier, 179, when calculating the mean of the monthly sales? Explain your answer.

   ....

   ....

   ....

MANIPULATION IN SUMMARY REPORTS ON DATA

The mode, median and mean each highlight different bits of information about the same set of data. They can be very different from one another, depending on the kind of data set you have.

Sometimes people choose the statistic that does not show the typical values, but rather the value that works best for them.

1. Thivha sells restored furniture. He reports that he usually sells seven items per week, and that he has the data to prove it. The receipts show that he made 52 sales over a period of eight weeks.

   (a) Can you tell from the data above whether Thivha is truthful about the sales?

   ....

   ....

   ....

   (b) You examined the receipts for the 8 weeks closely, and find the following number of sales per week:

   3, 4, 4, 4, 4, 5, 6, 22

   Determine the mode and median of the data set.

   ....

(c) Do you think the mode, median or mean is a better reflection of Thivha's typical sales figures per week? Explain your answer.

....

....

....

....

....

2. The following data shows the amount of pocket money that a group of learners receive per week.

R0	R0	R5	R10	R10	R10	R10	R20	R20	R50

(a) Determine the mode, median, mean and range of the data set.

....

....

....

....

(b) The teenager who receives R5 a week wants to convince her parents to give her more money. Which of the summary statistics would she use? Explain your answer.

....

....

(c) Which summary statistic do you think best represents the weekly pocket money in this group of learners? Explain your answer.

....

....

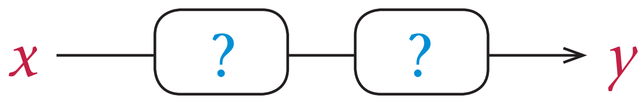
....

....

Chapter 9
Functions and relationships

In this chapter, you will use formulae to calculate output values for given input values. You will also learn to represent functions in different forms of representation: in words, with a flow diagram, a table and a formula. You will also use your knowledge of formulae to solve some problems.

$$y = 3(x + 2)$$



x	–2	–1	0	1	2
y	?	?	?	?	?

add 2 to the input number
and then multiply the answer by 3

9 Functions and relationships

9.1 Calculating output values

FORMULAE AND TABLES

The statement $y = 2x + 6$ can be true for any value of x, provided one chooses the appropriate value of y. The statement is true for certain combinations of values of x and y. Such a statement is called a **formula**.

A formula is a description of how the values of a dependent variable can be calculated for any given values of the other variable(s) on which it depends.

1. Which of the following are formulae for the function illustrated in the table?

   A. $y = 15x$ B. $y = -5x + 20$ C. $y = 5(20 - x)$ D. $y = 5x + 10$

x	1	2	3	4	5	6
y	15	10	5	0	–5	–10

....

....

2. For each of the tables below determine which of the following formulae could have been used to complete the table. The letter symbol x is used to represent the input numbers and the symbol y represents the output numbers.

   A. $y = x^2$ B. $y = 10x$ C. $y = 10x - 1$
   D. $y = x^2 + 2$ E. $y = 5x + 2$ F. $y = -5x + 2$
   G. $y = 3^x$ H. $y = 3^{x+1}$

(a)

Input value	1	4	11	30	40	60
Output value	7	22	57	152	202	302

....

(b)

Input value	1	6	9	12	18	20
Output value	1	36	81	144	324	400

....

(c)

Input value	1	6	9	12	18	20
Output value	3	38	83	146	326	402

....

(d)

Input value	3	11	19	27	45	70
Output value	30	110	190	270	450	700

....

(e)

Input value	3	11	19	27	45	70
Output value	29	109	189	269	449	699

....

(f)

Input value	1	2	3	4	5	6
Output value	3	9	27	81	243	729

....

9.2 Different forms of representation

FLOW DIAGRAMS, TABLES, WORDS AND FORMULAE

1. This question is about the relationship between two variables. Some information about the relationship is given in the flow diagram below.

   input number → $\times 3$ → $+ 2$ → output number

   (a) Use the instructions in the flow diagram to complete the table.

Input value	1	2	3	4	5	10	23	50	86
Output value									

   (b) Describe by means of a formula how to calculate the output number for any input number. (Let x represent the input numbers and y the output numbers.)

....

   (c) Describe verbally how to calculate the output number for any input number.

....

....

When there is only one output number for any input number, the relationship between the two variables is called a **function**.

(d) What input number will make the statement $3x + 2 = 71$ true? ....

(e) What input number will make the statement $3x + 2 = 260$ true? ....

2. Some information about the relation between output and input values in a certain function is given in the flow diagram.

input number —[+ 2]—[× 3]→ output number

(a) Use the flow diagram to complete the table below.

Input value	1	2	3	4	5			50	86
Output value						36	75		

(b) Describe by means of a formula how the input and output numbers are related. Use the letter y for the output numbers and x for the input numbers.

....

(c) Give a verbal description of how the input and output numbers are related.

....

(d) Themba wrote the formula $y = (x + 2)3$ to describe how the input and output numbers are related. Is Themba correct? Explain.

....

....

3. A certain function g is represented by means of the formula $y = 2(x - 4)$.

(a) Complete the table for g.

Input value	1	2	3	4	5	6	14	44	54
Output value				0	2	4	20	80	100

(b) Complete the flow diagram for g (fill in the operators):

input value —[ ]—[ ]→ output value

4. (a) Complete the table for the relation given by the formula $y = 2x - 4$.

Input value	1	2	3	4	5	6	14	44	54
Output value									

(b) Complete the flow diagram (fill in the operators):

input value —[ ]—[ ]→ output value

(c) Give a verbal description of how to complete the table.

....

Maths2_Gr8_LB_Book.indb 141 2014/09/04 10:39:24 AM

5. Complete the table.

<table>
<tr><th>Formula</th><th>Flow diagram</th><th>Table</th><th>Verbal description</th></tr>
<tr><td>$y = 4x$</td><td></td><td><table><tr><td>x</td><td>0</td><td>3,5</td><td>7</td><td>0,3</td></tr><tr><td>y</td><td>0</td><td>14</td><td></td><td></td></tr></table></td><td></td></tr>
<tr><td></td><td></td><td><table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table></td><td></td></tr>
<tr><td></td><td></td><td><table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>7</td><td>11</td><td>15</td><td>19</td></tr></table></td><td>Multiply the input number by 4 then subtract 1.</td></tr>
<tr><td>$y = 2(x + 1)$</td><td>→ [+ 1] → [× 2] →</td><td><table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td></tr><tr><td>y</td><td>−2</td><td>0</td><td>2</td><td>4</td></tr></table></td><td></td></tr>
<tr><td>$y = 2x + 2$</td><td>→ [× 2] → [+ 2] →</td><td><table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td></tr></table></td><td>Multiply the input number by 2 then add 2.</td></tr>
<tr><td>$y = 2x + 1$</td><td>→ [ ] → [ ] →</td><td><table><tr><td>x</td><td>−2</td><td>−1</td><td>0</td><td>1</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td></tr></table></td><td>Multiply the input number by 2 then add 1.</td></tr>
</table>

In sections 9.1 and 9.2, you have used four different ways to represent functions, namely:

- a formula,
- a table,
- a flow diagram and
- a verbal representation.

Later this term you will also represent functions by using **coordinate graphs**.

9.3 Completing more tables

LOOKING AT DIFFERENT FORMULAE AT THE SAME TIME

1. Use the given formulae in each column to complete the table below. Some rows have been completed for you. You may use a calculator.

x	$y = 10x$	$y = 10x^2$	$y = 10^x$
–7	–70	490	0,0000001
–6		360	0,000001
–5		250	0,00001
–4		160	0,0001
–3		90	0,001
–2		40	0,01
–1		10	0,1
0	0	0	1
1	10	10	10
2	20	40	100
3	30		1 000
4	40		10 000
5	50		100 000
6	60	360	1 000 000
7	70	490	10 000 000

2. In each case choose the correct answer from those given in brackets.
As the input value increases by equal amounts (say from 1 to 2, 2 to 3, 3 to 4, and so on), the output value for:
(a) $y = 10x$ (increases by equal amounts/increases by greater and greater amounts)

....

(b) $y = 10x^2$ (increases by equal amounts/increases by greater amounts)

....

(c) $y = 10^x$ (increases by equal amounts/increases by greater and greater amounts)

....

3. (a) Complete the table below. Some examples have been done for you.

x	$y = -2x - 1$	$y = -2x$	$y = -2x + 1$
–4	$-2 \times -4 - 1 = 7$	$-2 \times -4 = 8$	$-2 \times -4 + 1 = 9$
–3	$-2 \times -3 - 1 = 5$	$-2 \times -3 = 6$	$-2 \times -3 + 1 = 7$
–2			
–1			
0			
1	$-2 \times 1 - 1 = -3$	$-2 \times 1 = -2$	$-2 \times 1 + 1 = -1$
2			
3			
4			

(b) Describe the relationships between the corresponding output numbers in the three columns.

....

....

....

4. (a) Complete the table.

x	$y = 2^{x-1}$	$y = 2^x$	$y = 2^{x+1}$
–1	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$
0	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$
1			
2			
3			
4			
5			
6			

(b) Describe the relationships between the corresponding output numbers in the three columns.

....

....

....

9.4 Solving some problems

LOOKING AT SOME SITUATIONS

1. The formula $y = 38 - 2x$ describes the relationship between y and x in a certain situation.
   (a) Complete the table for this situation.

x	10	5	15		8	
y				36		2

   (b) What is the output value if the input value is 8? ....
   (c) For what input value is the output value equal to 28? ....
   (d) Which input value makes the statement $36 = 38 - 2x$ true? ....

2. Consider rectangles which each have an area of 24 square units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $xy = 24$.
   Complete the table to represent this situation.

Length (x)					6	8	12	24
Breadth (y)	24	12	8	6				

3. Consider rectangles with a fixed perimeter of 24 units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $2(x + y) = 24$.
   Complete the table to represent this situation.

x	1	2	3	4	6					
y						5	4	3	2	1

4. The formula $b = 180° - \frac{360°}{n}$ gives the size b of each interior angle in degrees for a regular polygon with n sides (an n-gon).
   (a) Complete the table below.

Number of sides (n)	3	4	5	6	10	12
Angle size (b)						

   (b) What is the size of each interior angle of a regular polygon with 20 sides, and a regular polygon with 120 sides?

   ....

   (c) If each interior angle of a polygon is 150°, how many sides does it have?

   ....

5. As you may know, metals contract when temperatures are low and expand when temperatures are high. So, when engineers build bridges they always leave small gaps in the road between sections to allow for heat expansion.

For a certain bridge, engineers use the formula $y = 2{,}5 - 0{,}05x$ to determine the size of the gap for each 1 °C rise in temperature, where x is the temperature in °C.

(a) Complete the table below to show the size of the gap at different temperatures:

Temperature (°C)	3	4	10	15	25	30	35
Gap size (cm)							

(b) What is the size of the gap at each of the temperatures shown below?

0 °C   18 °C   –2 °C   50 °C

....

(c) At what temperature will the gap close completely? ....

6. The formula $y = 0{,}0075x^2$, where x is the speed in km per hour and y the distance in metres, is used to calculate the braking distance of a car travelling at a particular speed. Use a calculator for this question.

The braking distance is the distance required for a vehicle travelling at a certain speed to come to a complete stop after the brakes are applied.

Example

What is the braking distance if someone drives at 80 kilometres per hour?

On your scientific calculator you must punch in 0,0075 followed by × sign followed by (80) followed by x^2. The calculator will do the following:

$y = [0{,}0075 \times (80)^2] = (0{,}0075 \times 6\ 400) = 48$

∴ The braking distance is 48 m.

(a) What is the braking distance at a speed of 100 kilometres per hour?

....

(b) Calculate the braking distance at a speed of 60 kilometres per hour.

....

(c) Complete the table below. Give answers to two decimal places where necessary.

Speed in km/h	10	20	30	40	50	60	100
Braking distance in m							

Maths2_Gr8_LB_Book.indb 146 2014/09/04 10:39:25 AM

Refer to the table in question (c) to answer question (d) below.

(d) A car travels at a speed of 40 kilometres per hour. A sheep 7 m away on the side of the road suddenly runs onto the road. Will the car hit the sheep or will the driver be able to stop the car before it hits the sheep? Explain.

....

....

(e) A car travels at a speed of 90 km/h in an area that has school children crossing the road. What distance does the driver need to stop the car so that it does not hit the children?

....

....

7. (a) Use the formula $y = 1{,}06x$ to complete the table below.

x	100	200	300	400	500	1 000	5 000	10 000
y								

(b) What is the value of y if $x = 750$?

....

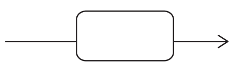
....

(c) What is the value of y if $x = 2\ 500$?

....

....

(d) Represent the formula $y = 1{,}06x$ by means of a flow diagram.



(e) If $y = 583$, what is the value of x?

....

....

(f) If $y = 954$, what is the value of x?

....

....

(g) The statement $1\ 060 = 1{,}06x$ is given. For what value of x is the statement true?

....

(h) The statement $530 = 1{,}06x$ is given. For what value of x is the statement true?

....

8. The formula $y = 0{,}1x + 5\ 000$ is given. What is the value of y in each case below?

(a) $x = 10$ ....

(b) $x = 100$ ....

(c) $x = 1\ 000$ ....

(d) $x = 10\ 000$ ....

9. The formula $y = 1{,}14x$ is used to calculate the price y of goods including VAT in rands, where x is the price in rands before VAT.
   (a) How much will you pay at the counter for goods that cost R38,00 without VAT, and for goods that cost R50,00 without VAT? You may use a calculator.

   ....

   ....

   (b) Complete the table for the prices of goods with VAT.

x	1	2	3	4	5	6	7	8	9
y									

10. Use your answers for question 9(b) to find the prices with VAT for goods with the following prices before VAT was added. Do not use a calculator at all in this question.
   (a) R40 .... (b) R400 ....
   (c) R70 .... (d) R470 ....

11. (a) An article costs R11,40 with VAT included. What is the price before VAT was added? ....
   (b) An article costs R342 with VAT included. What is the price before VAT was added? ....

12. Consider the function represented by $y = 75 - 0{,}1x$. What is the value of y if
   (a) $x = 0$? (b) $x = 750$?

   ....

   ....

   ....

   (c) Complete the table below for the function represented by $y = 75 - 0{,}1x$.

x	0	10	20	50	100	200	500	700	800
y									

   (d) For what value of x is $75 - 0{,}1x = 0$?

   ....

   ....

   (e) For what value of x is $75 - 0{,}1x = 100$?

   ....

   ....

Chapter 10
Algebraic equations

In this chapter, you will revise work you did in Grade 7 on equations. You will also learn how to solve equations by using additive and multiplicative inverses as well as properties of exponents. You will also substitute values in equations to generate tables of ordered pairs.

4	71	53	8	98	78	54
46	9	6	2	60	81	62
70	6	8	33	2	40	64
27	70	31	63	59	71	62
42	85	32	85	81	51	73
70	64	33	96	32	23	69
82	9	59	54	96	43	29
63	71	86	81	6	29	56
74	21	17	94	6	33	56
18	63	73	76	91	32	39
3	87	23	94	84	75	69
36	49	90	73	62	70	22
10	91	40	92	68	87	57
62	76	72	79	68	25	8
9	72	31	37	37	46	49
48	58	64	92	34	83	95
18	50	88	51	92	89	10
49	49	100	60	60	75	40

10 Algebraic equations

10.1 Revision

SETTING UP EQUATIONS TO DESCRIBE PROBLEM SITUATIONS

1. Farmer Moola has already planted 100 apple trees and 250 orange trees on his fruit farm. He decides to plant 20 more apple trees every day, as can be seen in the table below.

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	100	120	140	160					

He also decides to plant 10 orange trees a day, as shown below.

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	250	260	270	280					

(a) Write a rule for calculating the number of apple trees after x days. Write the rule in the form of a formula. Represent the number of trees with the letter symbol y.

....

(b) Write a formula for finding the number of orange trees after x days.

....

(c) How many orange trees are there on the 14th day?

....

(d) After how many days will Farmer Moola have 260 apple trees?

....

After how many days will farmer Moola have 1 000 apple trees in his orchard?

It will take some time to work this out by counting in twenties, and one can easily make a mistake and not even be aware of it. Another way to find the information is to figure out for which value of x it will be true that $100 + 20x = 1\,000$. To do this you may try different values for x until you find the value that makes $100 + 20x$ equal to 1 000. It is convenient to enter the results in a table as shown below. Anna first tried $x = 10$ and saw that 10 is far too small. She next tried $x = 100$ and it was far too big. She then tried 50.

Number of days (x)	10	100	50		
Number of apple trees (y)	300	2 100	1 100		

2. What number do you think Anna should try next, in her attempt to solve the equation $100 + 20x = 1\,000$?

....

3. How many days after he had 250 orange trees, will farmer Moola have 900 orange trees on his farm?

....

....

....

....

....

....

....

4. In 2004, there were 40 children at Lekker Dag Crèche. From 2005 onwards, the number of children dropped by about 5 children per year. Explain what each number and letter symbol in the formula $y = 40 - 5x$ may stand for.

....

....

....

5. Cool Crèche started with 20 children when it was opened in 2008. The number of children in Cool Crèche increases by 3 children every year. Explain what each letter symbol and number in the formula $y = 20 + 3x$ stands for in this situation.

....

....

....

6. Farmer Thuni already has 67 naartjie trees and 128 lemon trees on his fruit farm. He decides to plant 23 new naartjie trees and 17 new lemon trees every day during the planting season.
   (a) Which quantities in this situation change as time goes by, and which quantities remain the same?

   ....

   ....

   ....

Quantities that change are called **variables**, and are represented with letter symbols in formulae and equations. Quantities that do not change are called **constants**, and are represented by numbers in formulae and equations.

(b) How many naartjie trees and how many lemon trees will he have in total, 10 days after the planting season has started?

....

(c) Write formulae that can be used to calculate the total numbers of naartjie and lemon trees after any number of days during the planting season. Use letter symbols of your own choice to represent the variables.

....

(d) What information about the situation on farmer Thuni's farm can be obtained by solving the equation $67 + 23x = 500$?

....

(e) Set up an equation that can be used to find out how many days into the planting season it will be when farmer Thuni has 500 lemon trees. Use a letter symbol of your own choice to represent the unknown number of days.

....

SOLVING EQUATIONS BY INSPECTION

To **solve** an equation means to find the value(s) of the unknown for which an expression has a given value.

One method of solving an equation is to try different values of the variable until you find a value for which the expression is equal to the given value or for which the two expressions have the same value. This is called **solving by inspection**.

The value of the variable for which an expression is equal to a given value, or for which two expressions have the same value, is called the **solution** or the **root** of the equation.

1. In each case, determine whether the value of x given in brackets is a root or solution of the equation or not. Justify your answer, in other words say why you think the number is a root of the equation, or why not. In cases where the given number is *not* a root or solution, find the solution by trying other values.
   (a) $3x + 1 = 16$ $(x = 5)$ ....
   (b) $7x = 91$ $(x = 13)$ ....
   (c) $10x + 9 = 7x + 30$ $(x = 6)$ ....

....

Maths2_Gr8_LB_Book.indb 153 2014/09/04 10:39:26 AM

(d) $-10x - 1 = 29$ $(x = 3)$ ....

....

(e) $7 + 2x = 9$ $(x = 1)$ ....

2. Find the solution of each equation by inspection.

(a) $x - 1 = 0$ .... (b) $x + 1 = 0$ ....

(c) $1 + x = 0$ .... (d) $1 - x = 0$ ....

3. In each case, check whether the number in brackets makes the equation true. Explain your answer.

(a) $8 + x = 3$ $(x = 5)$ ....

(b) $8 + x = 3$ $(x = -5)$ ....

(c) $8 - x = 3$ $(x = 5)$ ....

(d) $8 - x = 3$ $(x = -5)$ ....

(e) $8 - x = 13$ $(x = -5)$ ....

(f) $8 - x = 13$ $(x = 5)$ ....

10.2 Solving equations

ADDITIVE AND MULTIPLICATIVE INVERSES

One way of thinking about the **additive inverse** of a number is to ask the question: What do I need to add to the given number to get 0?

1. What is the additive inverse of each of the following? Explain your answers.

(a) 5 .... (b) -5 ....

(c) 17 .... (d) 0,1 ....

(e) $\frac{5}{6}$ .... (f) $-2\frac{1}{4}$ ....

We can think of the **multiplicative inverse** of a number as asking the question: What do I need to multiply the number by to get 1?

2. What is the multiplicative inverse of each of the following? Explain.

(a) 5 ....

(b) –5 ....

(c) $\frac{5}{6}$ ....

(d) $\frac{1}{8}$ ....

You can solve the equation $2x + 5 = 45$ in the following way:

$2x + 5 = 45$		
$2x + 5 - 5 = 45 - 5$	Subtract 5 from both sides to have only the term in x	This step can also be understood as $5 + (-5) = 0$
$2x + 0 = 40$		
$\frac{2x}{2} = \frac{40}{2}$	Divide both sides by 2 to have x only	This step can also be understood as $2 \times \frac{1}{2} = 1$
$x = 20$		

3. Solve the equations below. Check that the value of x that you get is the solution.

(a) $5x + 2 = 32$

....

....

(b) $3x - 5 = -11$

....

....

(c) $5x = 40$

....

....

(d) $5x - 12 = 28$

....

....

(e) $\frac{3}{5}x = 15$

....

....

EXPONENTIAL EQUATIONS

Example: Solve the equation $2^x = 8$

Solution: $2^x = 2^3$ (Write 8 in terms of base 2, i.e. as a power of 2)

$x = 3$ (2 raised to the power of 3 is 8)

Solving exponential equations is the same as asking the question: To which power must the base be raised for the equation to be true?

1. Solve for x:

(a) $4^x = 64$

....

....

(b) $3^x = 27$

....

....

(c) $6^x = 216$

....

....

(d) $5^x = 125$ (e) $2^x = 32$ (f) $12^x = 144$

....

....

Another example: Solve the equation $2^{x+1} = 8$

Solution: $2^{x+1} = 2^3$

$x + 1 = 3$

$x = 2$

2. Solve for x:

(a) $4^{x+1} = 64$ (b) $3^{x-1} = 27$ (c) $2^{x+5} = 32$

....

....

....

....

10.3 Generating tables of ordered pairs

PRELIMINARY ACTIVITY

1. Complete the table below for the given values of x:

x	–3	–2	–1	0	1	2	3	6	10
$-3x + 2$									

2. What is the value of $15x + 3$ for

(a) $x = 2$? (b) x = multiplicative inverse of 15?

....

3. For what values of x are the following equations true?

(a) $15x + 3 = 33$ (b) $15x + 3 = 4$

....

....

....

In questions 1 and 2 you evaluated an expression for given values of x.

In question 3 you determined a value of x that makes the equation true. In other words, you solved for x.

In completing a table of values you may be confronted with questions similar to questions 1, 2 and 3.

FROM A FORMULA TO A TABLE OF VALUES

1. Complete each of the tables below for the given formula.

(a) $y = x$

x		–3	–2	–1	0	1	2		10	
y	–9							8		15

(b) $y = x + 2$

x		–3	–2	–1	0	1	2		10	
y	–5							8		15

(c) $y = x^3$

x		–3	–2	–1	0	1	2		6	
y	–216							125		1 000

2. Use the table in 1(c) to solve for x in each case below.

(a) $x^3 = -1$ .... (b) $x^3 = 8$ .... (c) $x^3 = 0$ ....

3. Complete the table for $y = 2x$.

x		–3	–2	–1	0	1	2		10	
y	–14							8		26

4. Use the table in question 3 to answer the following questions:

(a) What value of x makes the equation $2x = 20$ true? ....

(b) For what value of x is $2x = 0$? ....

5. Complete the table for $y = -x - 2$.

x		–3,5	–2	–1	0	1,2	2		6,9	
y	5							–8		–15

6. For what values of x are the following equations true?

(a) $-x - 2 = 0$ (b) $-x - 2 = 5$ (c) $-x - 2 = -4$

....

7. Complete the table for $y = x^2$.

x	–4	–3	–2	–1		1		3	4	13
y					0		4			169

8. Refer to the table in question 7 to answer the questions that follow:

(a) Which different values of x make the equation $x^2 = 16$ true?

....

(b) Solve for $x^2 = 9$.

....

(c) Solve for $x^2 = 169$.

....

(d) What is the solution of $x^2 - 1 = 3$?

....

9. Some tables of ordered pairs are given below. For each table, find out which of the following formulae was used to make the table. Write the correct formula above each table.

$y = -5x - 2$ $\quad$ $y = 5x + 2$ $\quad$ $y = 2x + 5$ $\quad$ $y = 2x - 5$

$y = 2x - 5$ $\quad$ $y = -5x + 2$ $\quad$ $y = -3x + 2$ $\quad$ $y = 3x + 2$

(a) ....

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–3	–1	1	3	5	7	9	11	13	15

(b) ....

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–18	–13	–8	–3	2	7	12	17	22	27

(c) ....

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–10	–7	–4	–1	2	5	8	11	14	17

(d) ....

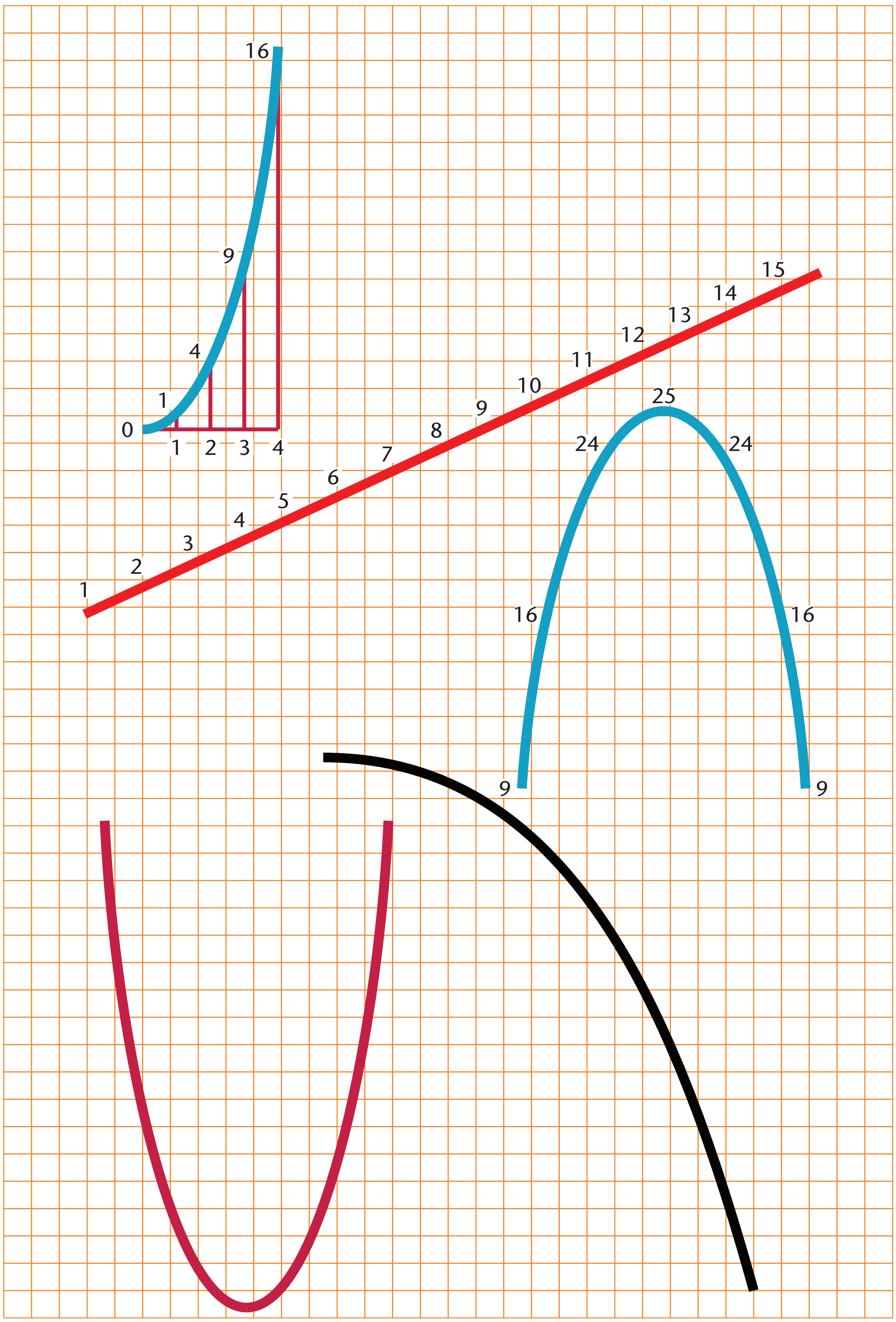
x	–4	–3	–2	–1	0	1	2	3	4	5
y	18	13	8	3	–2	–7	–12	–17	–22	–27

(e) ....

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–13	–11	–9	–7	–5	–3	–1	1	3	5

CHAPTER 11
Graphs

In this chapter, you will specifically deal with global graphs. These graphs show visually how variables vary, focusing on trends rather than detailed readings.



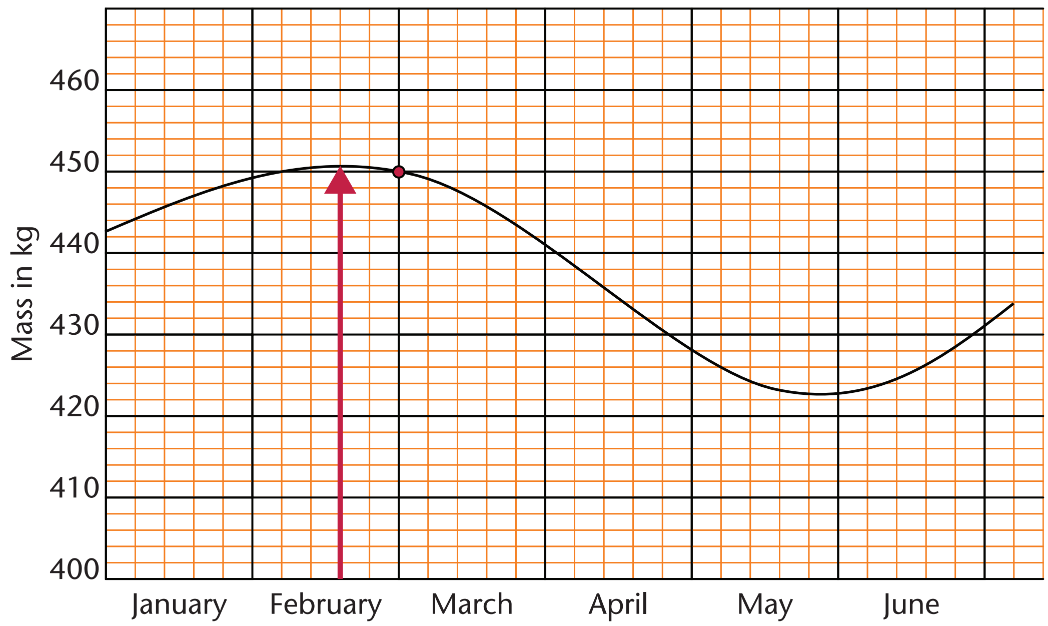
Maths2_Gr8_LB_Book.indb 160 2014/09/04 10:39:28 AM

11 Graphs

11.1 What we can tell with graphs

INTERPRETING GRAPHS

1. Mrs Maleka is a dairy farmer. She cares for her cows and weighs all of them daily. Here is a graph of one cow's mass in kilograms over a period of 6 months. At the end of February, the mass of the cow was 450 kg, as shown by the red dot.



(a) The mass of the cow reached a maximum a few days after the middle of February, as shown by the red arrow on the graph. When, in the period shown on the graph, did the cow's mass reach a minimum?

....

(b) During most of February the cow weighed slightly more than 450 kg. During which month did the cow weigh less than 430 kg, for the whole month?

....

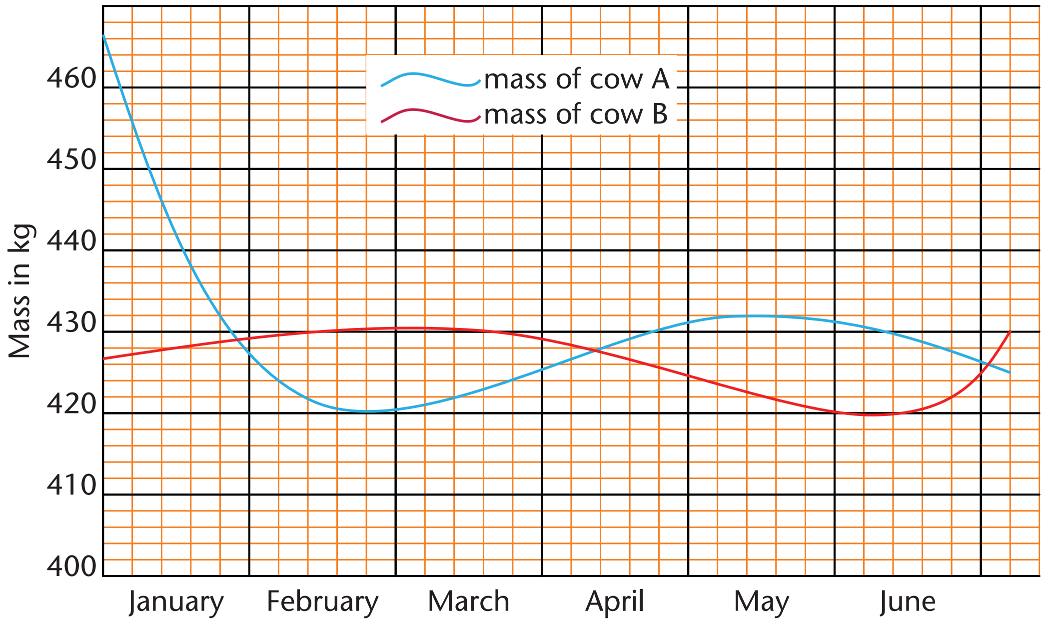
(c) All through the month of June, the mass of the cow increased. During which other month did the mass of the cow also increase, right through the month?

....

(d) During which months did the mass of the cow decrease right through the month?

....

2. The blue and red curves below are graphs that show how the mass of two cows varied over the same period of time.



(a) Which cow was the heaviest at the end of February? ....

(b) When was cow B heavier than cow A?

....

(c) During which months did the mass of cow A decrease for the whole month?

....

(d) When did cow A's mass start to increase again?

....

(e) During what month did cow B's mass begin to decrease while cow A's mass increased for that whole month?

....

(f) When did cow A's mass catch up with cow B's mass again?

....

(g) When did cow A stop gaining weight and start losing weight again?

....

(h) When did cow B's mass catch up with cow A's mass again?

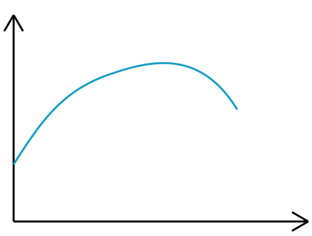
....

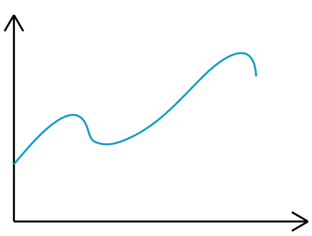
3. A traffic department keeps track of the traffic density on different roads. Two traffic officers are posted somewhere along each main road and they count and record the number of cars that pass in each direction during each 15 minute interval. They use tally marks to do this, as you can see in the example below.

	//// 𝍸 𝍸	/// 𝍸 𝍸 𝍸 𝍸	𝍸 // 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 //// 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	/ 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 /// 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	14	23	37	59	71	48

Which of the graphs below do you think is the best representation of the above data on traffic flow?

....

A


B


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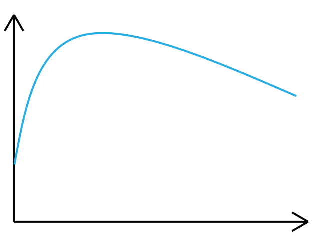

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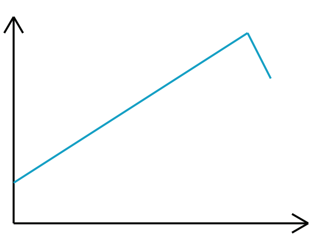

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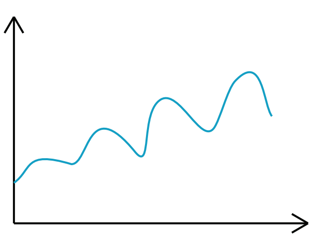

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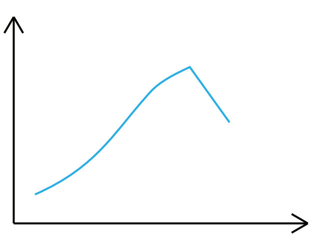

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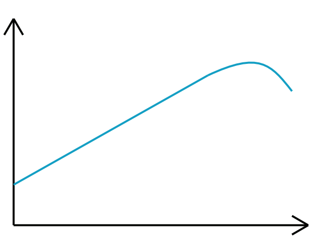

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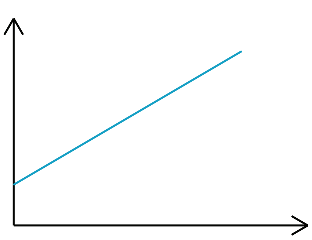

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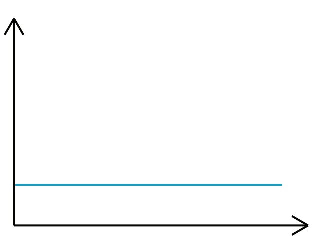


4. Which of the graphs on the previous page is the best representation of each of these traffic flow reports?

(a)

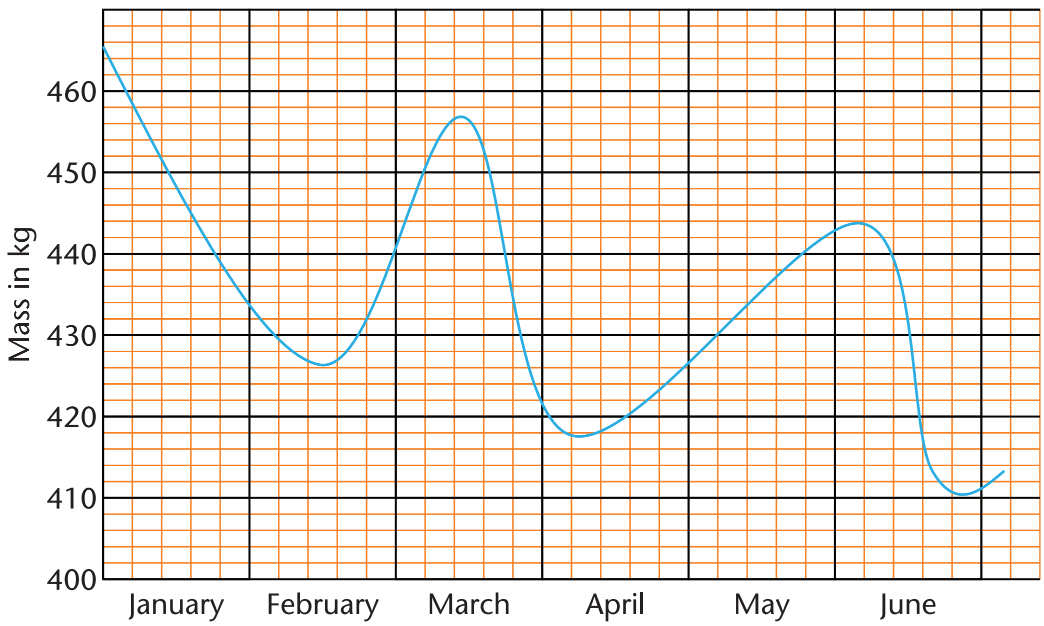
Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	53	64	75	86	75

(b)

Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	123	158	147	136	124

....

5. Study this graph for another cow.



(a) During which periods did the cow lose weight?

....

(b) During which of these periods did the cow lose weight more slowly?

....

(c) During which of the periods did the cow lose weight most rapidly?

....

(d) Compare the two periods when the cow gained weight.

....

....

Maths2_Gr8_LB_Book.indb 164 2014/09/04 10:39:30 AM

(e) Is there anything else about the graph that may indicate that this cow has health problems?

....

....

....

HOW GRAPHS SHOW INCREASES AND DECREASES

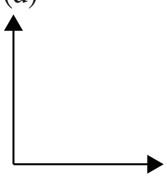
A graph on a system of coordinates shows the way in which one quantity (called the dependent variable) changes when another quantity (called the independent variable) increases. A quantity can change in different ways:

- It can increase or decrease.
- It can increase at a constant rate, for example the total amount saved if the same amount is saved every week or month.
- It can decrease at a constant rate, for example the length of a burning candle.
- It can increase (or decrease) at a varying rate, for example the increase in the area of a square as the side length increases.

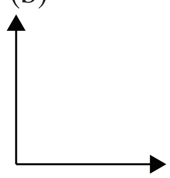
When a quantity increases or decreases at a **constant rate**, it is called **linear** change or variation, and the graph is a **straight line**. When the rate of change is **not constant**, it is called **non-linear** change, and the graph is **curved**. If there is no change in the output variable, the graph is a horizontal straight line.

1. Draw a graph to match each of the following descriptions.
   (a) The quantity increases, and increases more rapidly as time progresses.
   (b) The quantity first increases slowly at a constant rate, and then increases at a faster constant rate.
   (c) The quantity decreases faster and faster.
   (d) The quantity increases, and the rate of increase gradually diminishes.

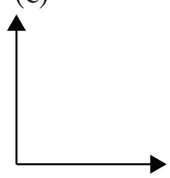
(a)



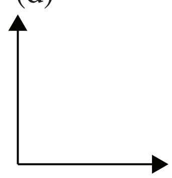
(b)



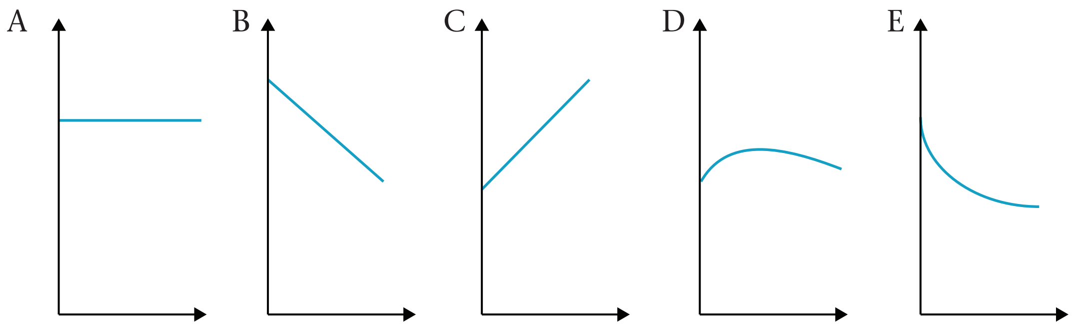
(c)



(d)



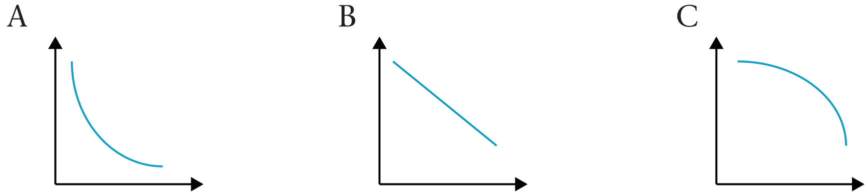
2. Statements and graphs about patterns of change in the petrol price per litre over a period are given below. Match each statement with the appropriate graph given below. Time is represented on the horizontal axis in all these graphs, and petrol price on the vertical axis.
   (a) The price did not change. ....
   (b) The price rose at a constant rate. ....
   (c) The price decreased at a constant rate. ....
   (d) The price dropped very fast at first and then at a slower rate. ....
   (e) The price rose at a decreasing rate up to a point and then started to drop at an increasing rate. ....



3. Complete the table below in respect of the graphs in question 2.

Graph	Represents a linear or non-linear relation	Reason
A		
B		
C		
D		
E		

4. (a) Which graph below represents a quantity that decreases at a constant rate? ......
   (b) Which graph represents a quantity that decreases at an increasing rate? ....
   (c) Which graph represents a quantity that decreases at a decreasing rate? ....



11.2 More features of graphs

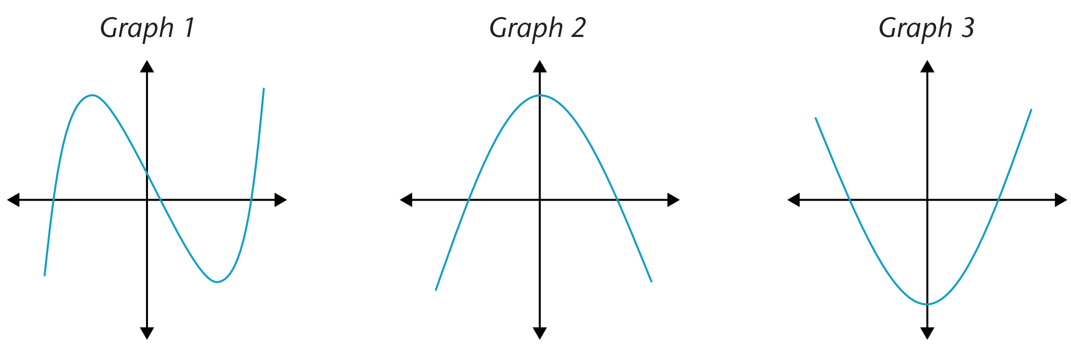
LOCAL MAXIMUM AND MINIMUM VALUES

A graph has a **maximum value** when it changes from increasing to decreasing.

A graph has a **minimum value** when it changes from decreasing to increasing.

A graph can have more than one minimum or maximum value.

1. Consider the graphs below. Describe how the dependent variable behaves in each case by indicating which graph corresponds to which description.
   (a) The variable has a maximum value because it changes from increasing to decreasing. ....
   (b) The variable has a minimum value because it changes from decreasing to increasing. ....
   (c) The variable has a maximum value as well as a minimum value because it changes from increasing to decreasing and then from decreasing to increasing. ....



2. On the next page, draw graphs that match the descriptions given below.
   (a) A quantity changes in non-linear fashion, at one stage switching from decreasing to increasing and then to decreasing again.
   (b) A quantity changes from increasing at a constant rate to decreasing at a constant rate and then becomes constant.

(a) (b)

DISCRETE OR "CONTINUOUS"

Quantities can be counted, measured or calculated.

1. Which of the items in the list provided can you count, and which quantities need to be measured?
   (a) Number of bags of cement sold
   (b) Heights of learners in Grade 8
   (c) Times taken for athletes to complete a 400 m hurdles race during the Olympics
   (d) The number of sweets in various 500 g bags
   (e) The distance travelled by learners to school
   (f) Cars passing at a scholar patrol crossing
   (g) The cost of an exercise book in rands and cents
   (h) Temperature

   Write your answers in the table.

Can only be counted	Can only be measured

2. Say whether the following make sense. Explain.
   (a) 501,3 learners attended a rugby match played by the senior team.

   ....
   (b) The distance from school to the nearest shopping mall is 10,75 km.

   ....
   (c) 2 004,75 cans of cola were sold during a fundraising event.

   ....

Quantitative data is numerical data such as your marks in a Mathematics test. Quantities that can be counted are sometimes said to be **discrete**: they do not allow values in between any two consecutive values. You cannot have 2,6 people for example. Quantities that allow many values between any two values are sometimes said to be **continuous**.

The terms "discrete" and "continuous" are used in different meanings than these in formal mathematics.

11.3 Drawing graphs

DRAWING GLOBAL GRAPHS

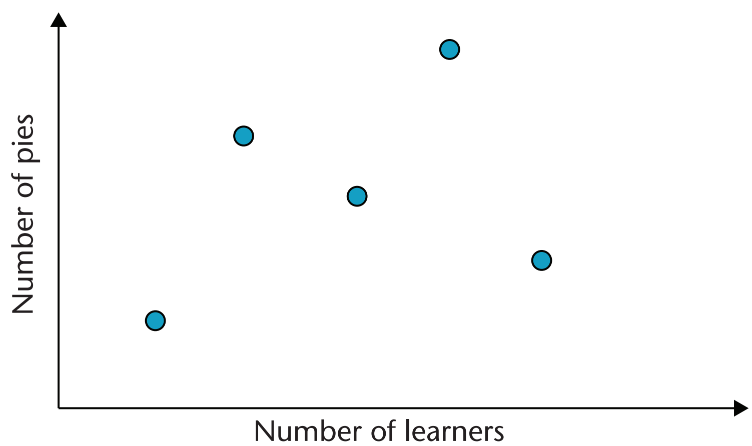
When we draw a graph of continuous data, it is a solid line or curve.

The graph of discrete data is a set of distinct points.

Consider the situations below.

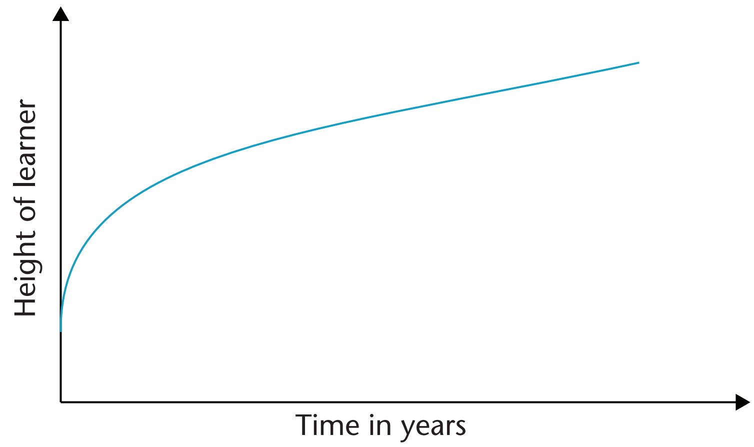
Situation 1

Number of pies bought by learners during a school week



Situation 2

Graph showing the height of a learner over a period of time



1. (a) What type of data is graphed in situation 1?

....

(b) What type of data is graphed in situation 2?

....

(c) Why do you think the graph in situation 2 is a solid line?

....

(d) Why are the points in situation 1 not joined?

....

2. Draw a rough graph for each of these situations. Use the spaces below and on the next page.
   (a) The height of a young tree and its age
   (b) The level of water in a dam over a period without any rain
   (c) The temperature under a tree over a period of 24 hours

(a)

(b)

(c)

GRAPHS OF ORDERED PAIRS

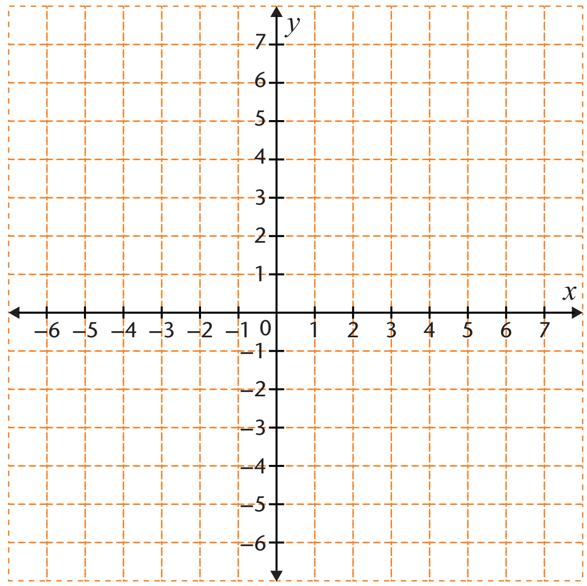
Input and output values can be written as a pair. The first number in a pair represents the input number and the second number represents the output number. We therefore say that the pair of numbers is ordered.

Making a graph of **ordered pairs** is another way to show how the input and output values are related.

When drawing a graph of ordered pairs, work as follows:

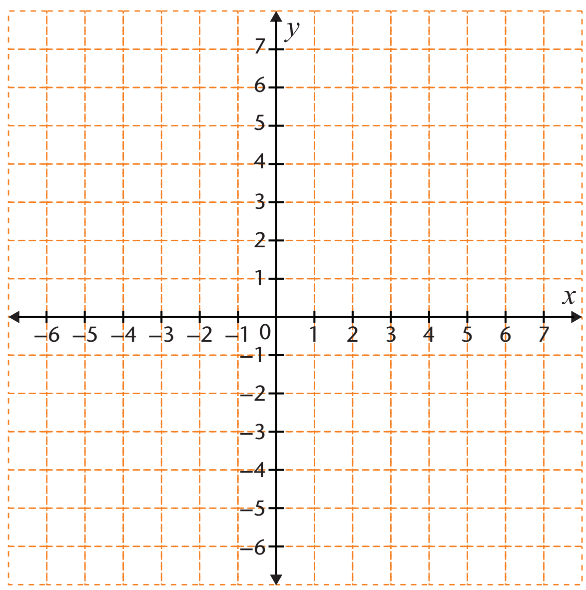
- First identify the input values (x) and output values (y). In most cases the input values will be given and the output values are calculated using the formula given.
- The output values are written on the y-axis (the vertical axis) and the input values are written on the x-axis (the horizontal axis).
- Plot the ordered pair. Suppose the ordered pair is (3; 6). To plot this pair put your finger on the number 3 on the x-axis and another finger on the number 6 on the y-axis. Move the finger on the number 3 in a line straight up and move the finger on the number 6 straight across. Where your two fingers meet, make a point. You can describe this point with the ordered pair (3; 6).

1. Plot the ordered pairs given below:
   (a) A(0; 3)
   (b) B(3; 0)
   (c) C(–2; 1)
   (d) D(4; –4)
   (e) E(–3; –2)



2. (a) Complete the table below for $y = x + 3$.

x	y	$(x; y)$
–4		
–3		
–2	1	(–2; 1)
–1		
0		
1	4	(1; 4)
2		
3		
4		

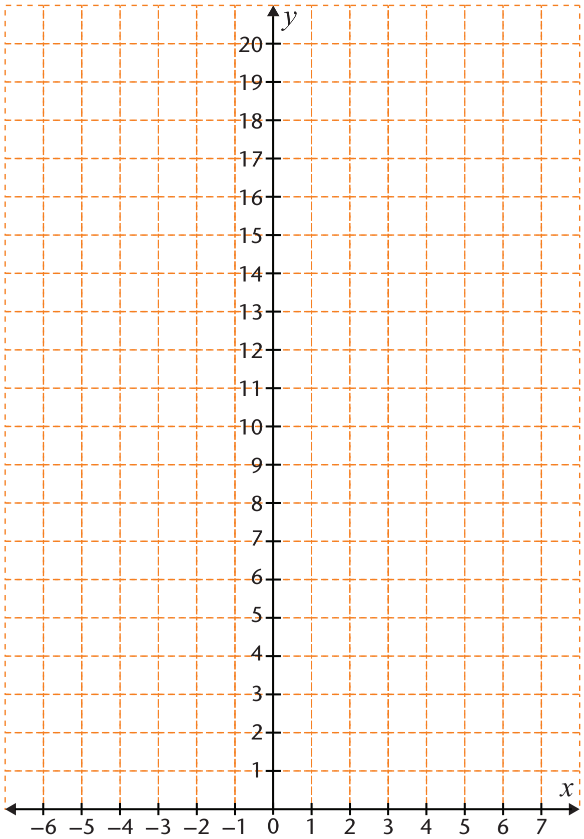


   (b) Plot the ordered pairs on the given coordinate system.
   (c) Join the points to form a graph.
   (d) The ordered pair (1; 6) is not on the graph because when we substitute the value of x ($x = 1$) in the formula $y = x + 3$ we get 4 instead of 6. [$y = 1 + 3 = 4$]
   Is the ordered pair (100; 103) on the graph? Explain.

. . . .

3. (a) Complete the table below for the formula $y = x^2 + 3$.

x	y	$(x; y)$
–4		
–3		
–2	7	(–2; 7)
–1		
0		
1	4	(1; 4)
2		
3		
4		



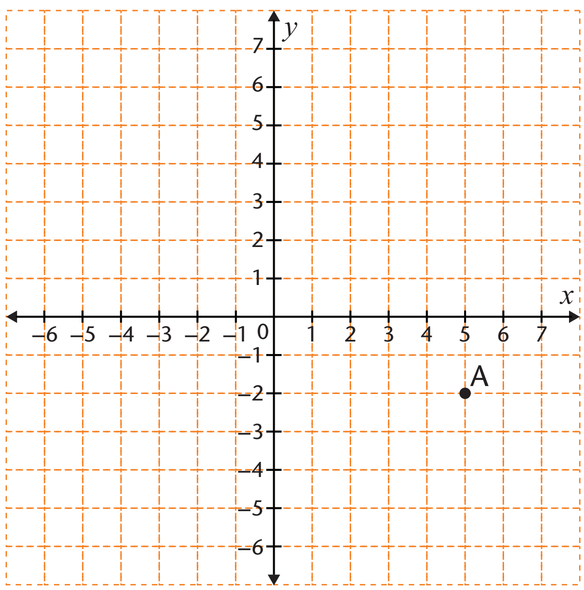
(b) Plot the coordinates on the axis system on the right. Join the points to form a graph.

(c) Is the point (10; 103) on the graph? Explain.

....

4. (a) Complete the table below for the formula $y = -x + 3$.

x	y	$(x; y)$
–4		
–3		
–2	5	(–2; 5)
–1		
0		
1	2	(1; 2)
2		
3		
4		



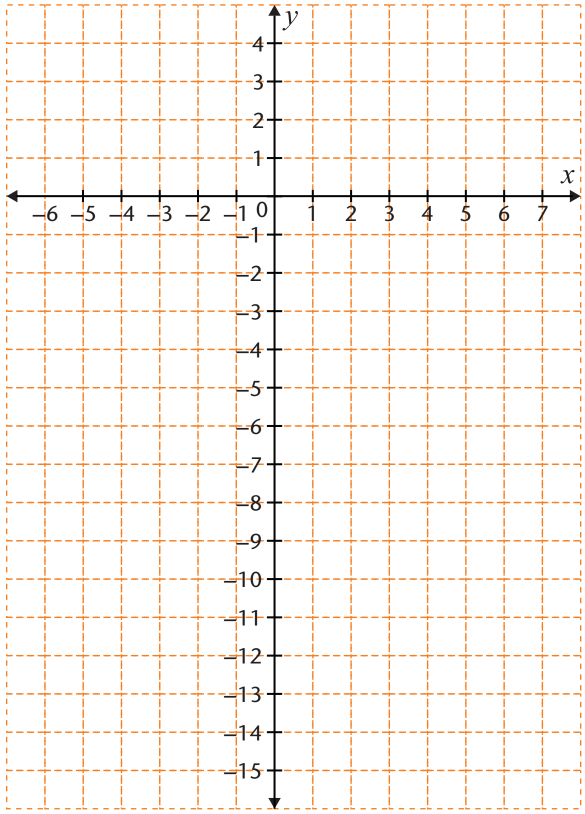
(b) Plot the ordered pairs on the axis system.

(c) Join the points to form a graph.

(d) What are the values of the ordered pair A on the graph? ....

5. (a) Complete the table below for the formula $y = -x^2 + 3$.

x	y	$(x; y)$
–4		
–3		
–2	–1	(–2; –1)
–1		
0		
1	2	(1; 2)
2		
3		
4		

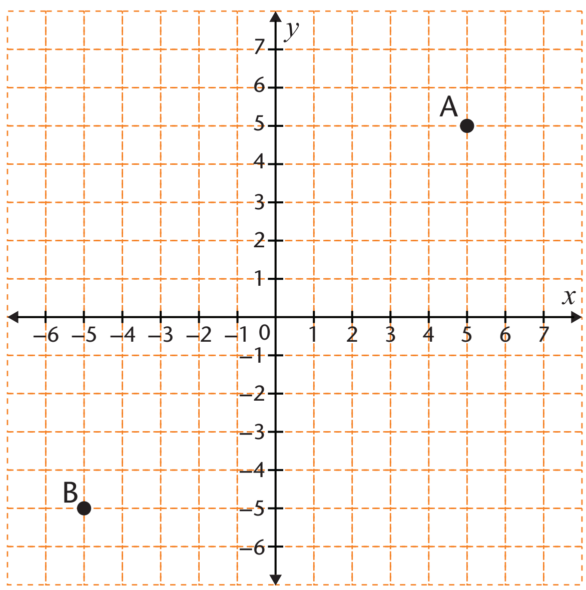


(b) Plot the ordered pairs on the axis system.

(c) Join the points to form a graph.

6. (a) Complete the table below for the formula $y = x$.

x	y	$(x; y)$
–4		
–3		
–2	–2	(–2; –2)
–1		
0		
1	1	(1; 1)
2		
3		
4		



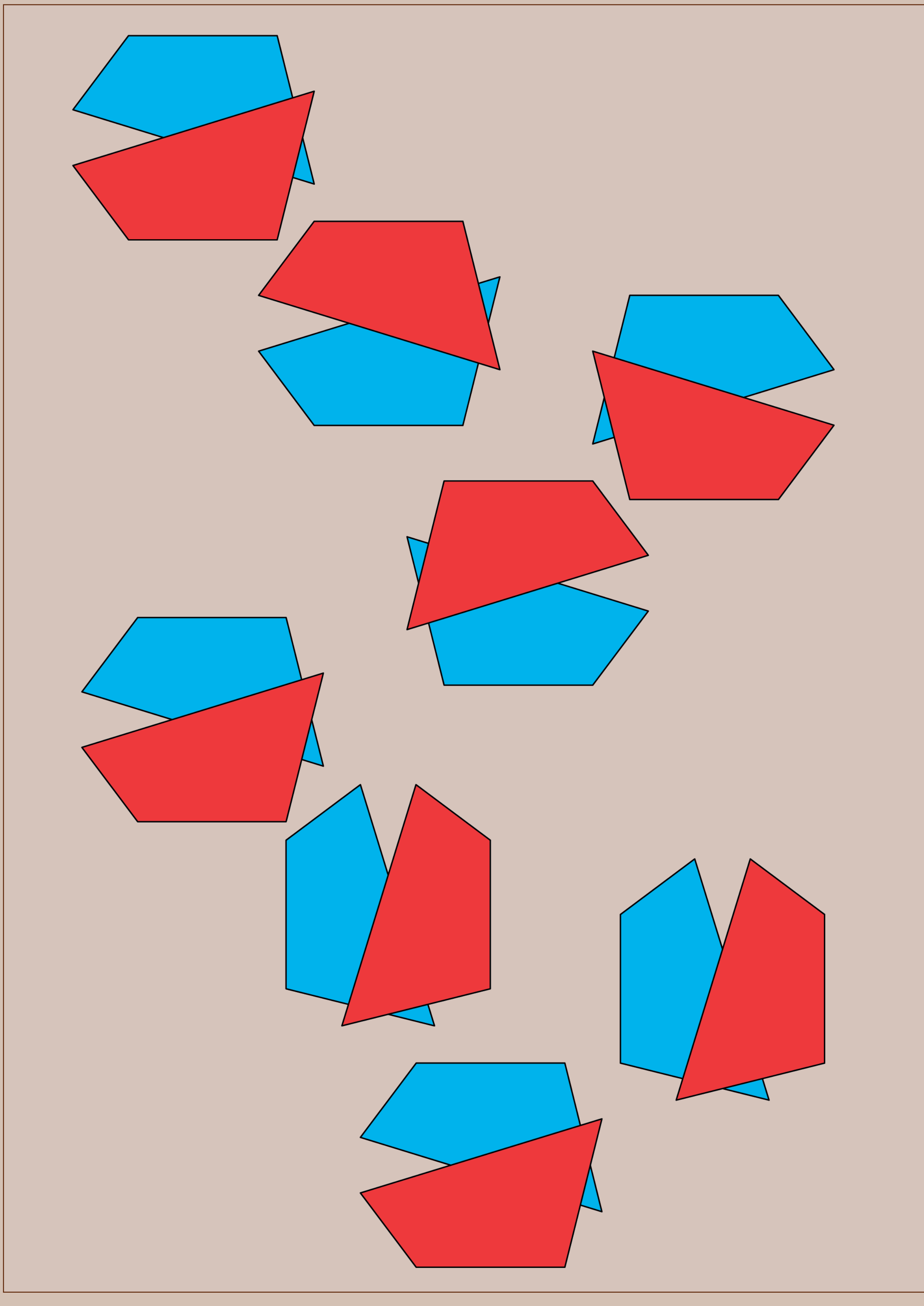
(b) Plot the ordered pairs on the axis system.

(c) Join the points to form a graph.

(d) Write down the values of the ordered pairs A and B on the graph.

Chapter 12
Transformation geometry

In previous grades, you learnt about translating, reflecting and rotating geometric figures. These changes in the positions of figures are types of transformations. You will now learn how to plot transformations on a coordinate system. Here, you will focus on the change in the coordinates of points and geometric figures on the coordinate system. You will also revise how to enlarge and reduce figures, and investigate in more detail how the sides of enlarged and reduced figures must be in proportion. Then you will explore how enlarging or reducing a figure affects the sizes of its perimeter and area.



12 Transformation geometry

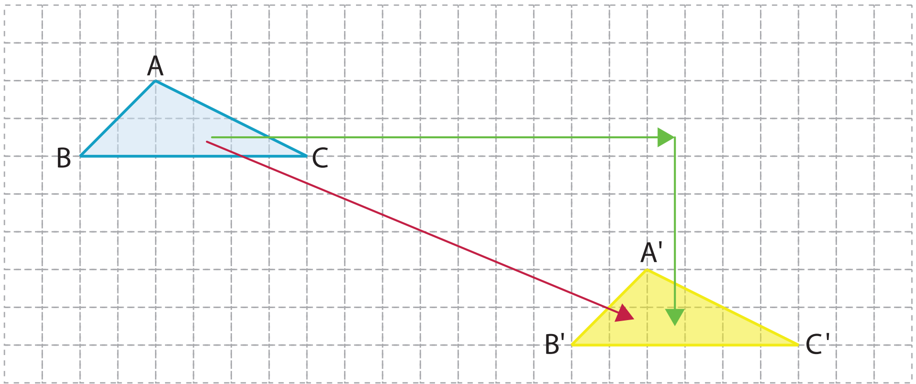
12.1 Transformations and coordinate systems

WHAT ARE TRANSFORMATIONS?

A figure can be moved from one position to another on a flat surface by **sliding** (translating), **turning** (rotating) or **flipping** it over (reflecting), or by a combination of such movements. These and other kinds of movements are also called **transformations**.

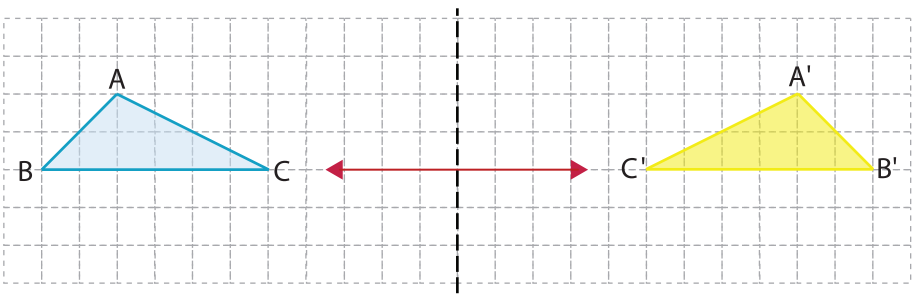
A slide, also called a ***translation***

A slide can also be performed in steps, as indicated by the green arrows.



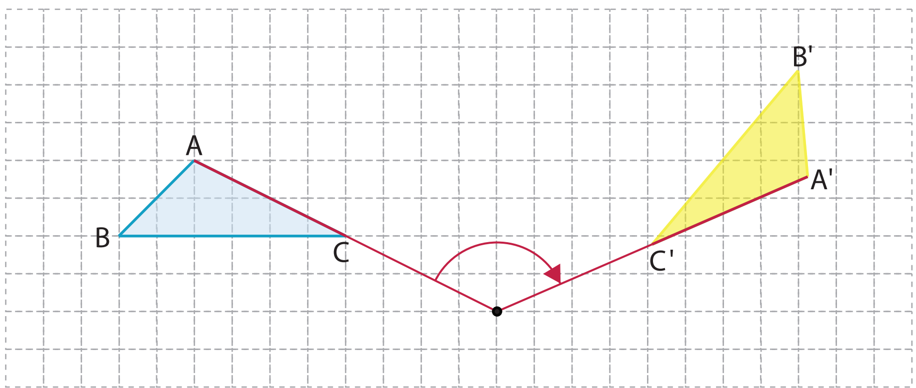
A flip-over, also called a ***reflection***

You may also think of folding the paper over on the dotted line.



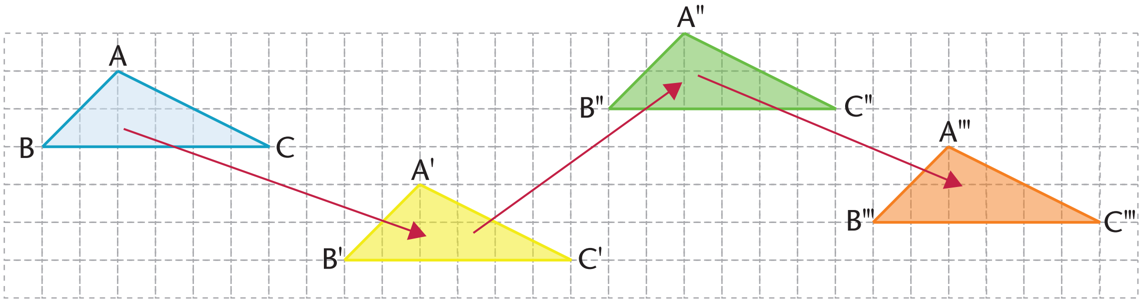
A swing or turn, also called a ***rotation***

The object is swung (rotated) clockwise or anticlockwise around a point called the **centre of rotation**. It is as if you hold the object on a string.



In its new position, the figure is called the **image** of the original figure. In the diagrams above, the original figures are blue and the images are yellow. Slides, turns and flips do not change the shape or size of a figure. Hence, in these transformations, the original figure and its image are always congruent.

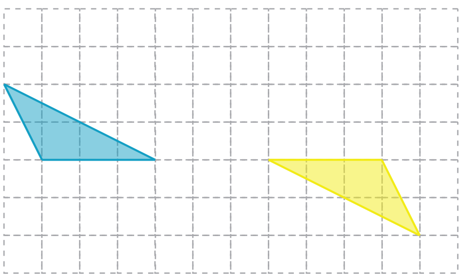
To name the image, we use the same letters as in the original figure, but we add the prime symbol (') after each letter. For example, the image of ΔABC is ΔA'B'C'. If there is a second image, we add two prime symbols, for example ΔA"B"C". If there is a third image, we use three prime symbols, for example ΔA'"B'"C'", and so on.



The grid in the background makes it possible to describe the different positions of the figure clearly. To do that, a **system of axes** can be drawn on the grid to form a **coordinate system**, as you will see on the next page. But first, answer the question below.

> A **coordinate system** consists of numbered horizontal and vertical lines that are used to describe position.

In each case, state whether the triangle was translated, reflected or rotated.

1. 

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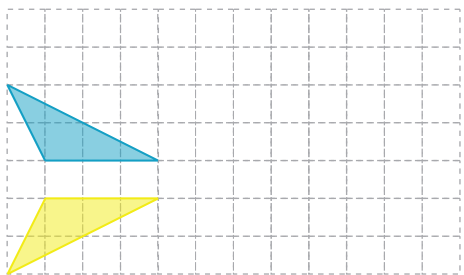
2. 

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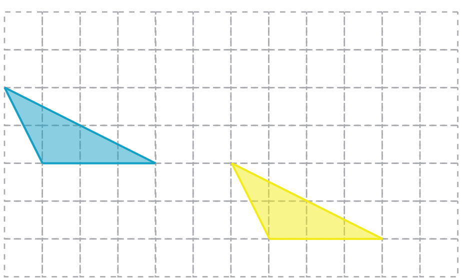
3. 

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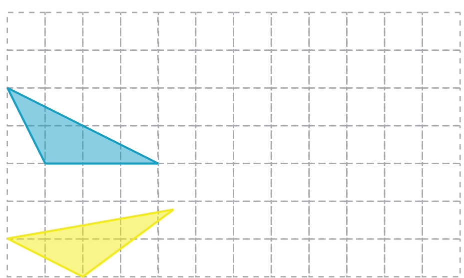
4. 

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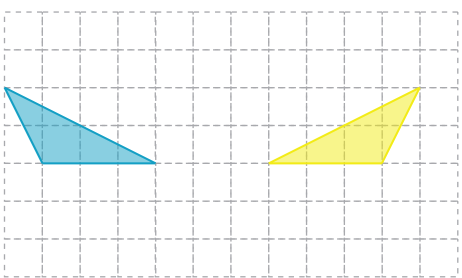
5. 

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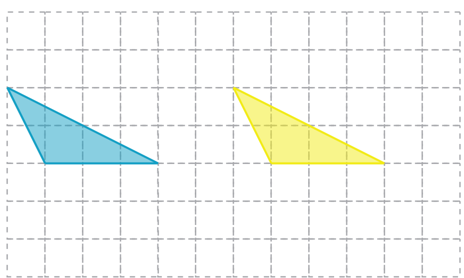
6. 

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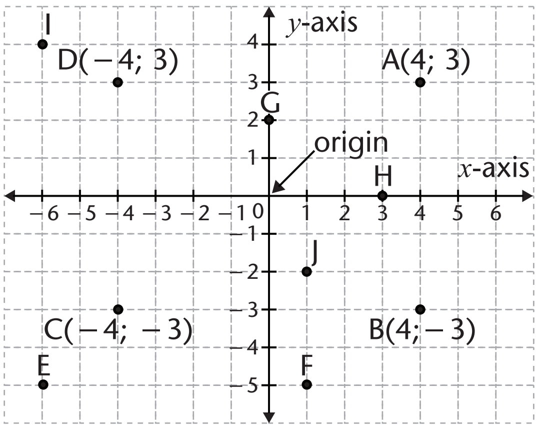
COORDINATE SYSTEMS

The position of any point on a system of coordinates can be described by two numbers, as demonstrated below for the points A, B, C and D.



In honour of the French mathematician Descartes who invented it, a coordinate system is also called *a system of Cartesian coordinates.*

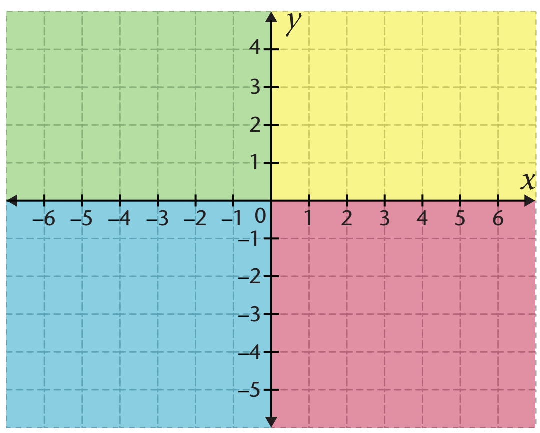
The horizontal axis on the coordinate system is called the *x*-axis and the vertical axis is called the *y*-axis. The ordered pair (4; 3) indicates that the value of the *x*-coordinate is 4 and the value of the *y*-coordinate is 3. A coordinate system is divided into four sections called **quadrants**.

1. What are the coordinates of each of the following points on the above grid?

   E .... F .... G ....

   H .... I .... J ....

The first quadrant is coloured yellow on the system on the right, the second quadrant green, the third quadrant blue and the fourth quadrant pink.



2. Mark the following points on the coloured coordinate system.

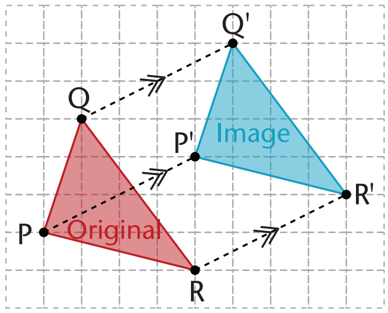
   A(5; 2) B(−4; 3)
   C(−5; 1) D(−3; −3)
   E(−6; −2) F(2; −3)
   G(5; −2) H(4; −6)

3. (a) In which quadrant are both coordinates positive? ....
   (b) In which quadrant are both coordinates negative? ....
   (c) In which quadrant is only the *x*-coordinate negative? ....
   (d) In which quadrant is only the *y*-coordinate negative? ....

12.2 Translation on the coordinate system

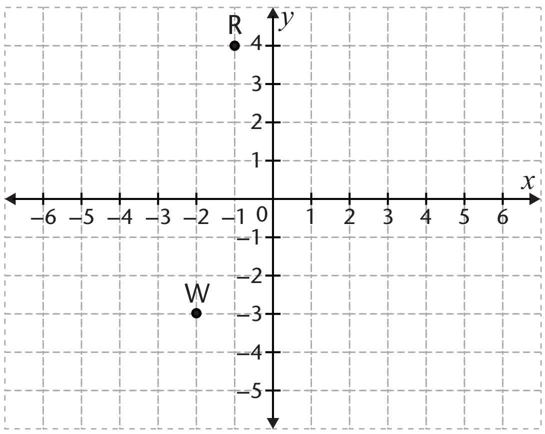
Revise the **properties of translation** from Grade 7:

- The line segments that connect any point in the original figure to its image are all equal in length. In the diagram: PP' = RR' = QQ'
- The line segments that connect any original point in the figure to its image are all parallel. In the diagram: PP'||RR'||QQ'
- When a figure is translated, its shape and size do not change. The original and its image are congruent.



TRANSLATING POINTS ON THE COORDINATE SYSTEM

1. Plot the image of each of the following translations.
   (a) R is translated 3 units down to R'.
   (b) R' is translated 4 units to the left, to R''.
   (c) W is translated 5 units to the right, to W'.
   (d) W' is translated 6 units up, to W''.



2. (a) Write down the coordinates of points A, B and C.

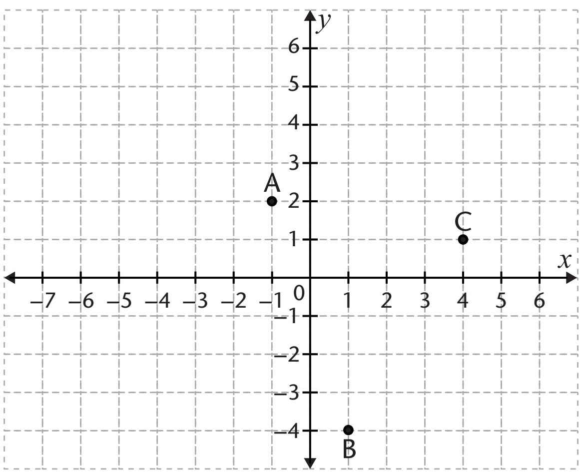
   ....

   (b) Translate A, B and C 6 units to the left and 4 units up.
   (c) Write down the coordinates of points A', B' and C'.

   ....

   (d) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (e) Are ΔABC and ΔA'B'C' congruent?

   ....



TRANSLATING TRIANGLES ON THE COORDINATE SYSTEM

When you plot the transformation of a shape, first plot the images of the vertices of the shape and then join the image points to create the shape.

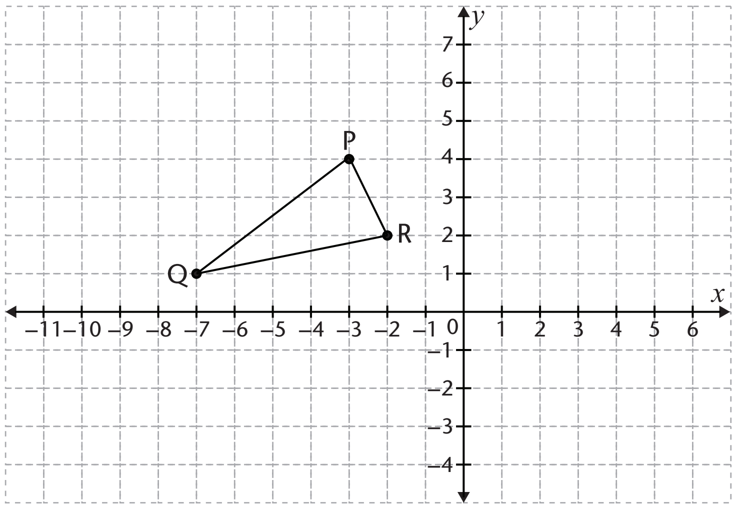
1. (a) Translate ΔPQR 6 units to the right and 2 units down. What are the coordinates of the vertices of ΔP'Q'R'?

   ....

   (b) Translate ΔPQR 4 units to the left and 3 units up. What are the coordinates of the vertices of ΔP''Q''R''?

   ....

   ....



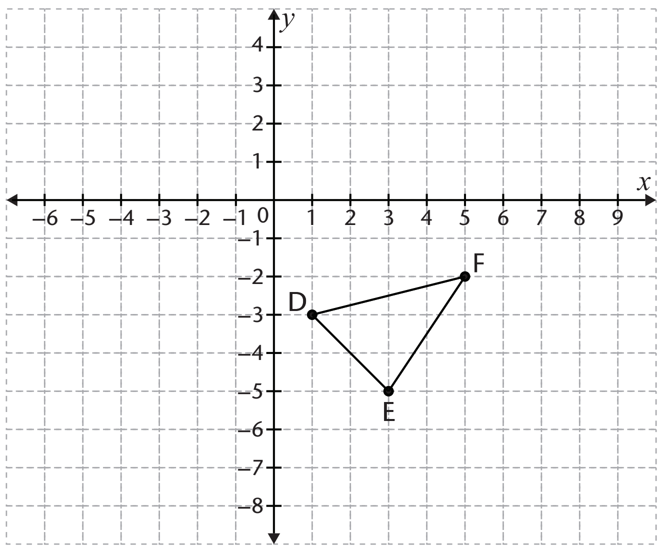
2. (a) Translate ΔDEF 4 units to the left and 2 units down. What are the coordinates of the vertices of ΔD'E'F'?

   ....

   ....

   (b) Translate ΔDEF 3 units to the right and 4 units up. What are the coordinates of the vertices of ΔD''E''F''?

   ....



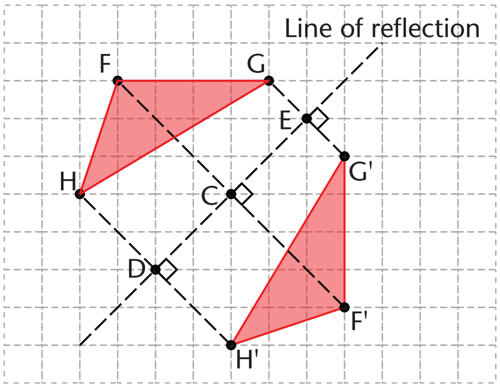
3. Write down the coordinates of the vertices of ΔKLM after each translation described in the table.

Vertices of triangle	Translated 5 units to the right and 2 units down	Translated 4 units to the left and 3 units down	Translated 2 units to the right and 3 units up
K(–1; 3)			
L(–2; –3)			
M(4; 0)			

12.3 Reflection on the coordinate system

Revise the **properties of reflection** from Grade 7:

- The image of ΔFGH lies on the opposite side of the **line of reflection** (mirror line).
- The distance from the original point to the line of reflection is the same as the distance from the image point to the line of reflection. In the diagram: GE = G'E; FC = F'C and HD = H'D.
- The line that connects the original point to its image point is always perpendicular (⊥) to the line of reflection. In the diagram: HH' ⊥ line of reflection, FF' ⊥ line of reflection and GG' ⊥ line of reflection.
- When a figure is reflected, the figure and its image are congruent.

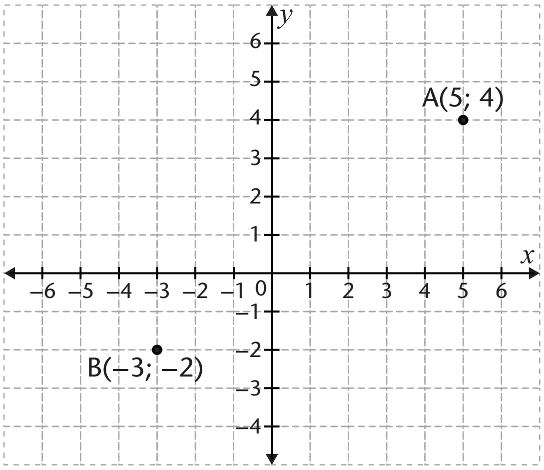


A line of reflection can run in any direction. This year, you will learn about reflections in the *x*-axis or in the *y*-axis only.

REFLECTING POINTS IN THE *x*-AXIS OR IN THE *y*-AXIS

Reflecting a point in the *x*-axis means that the *x*-axis is the line of reflection.

Reflecting a point in the *y*-axis means that the *y*-axis is the line of reflection.



1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system.

   (a) Reflect points A and B in the *x*-axis (horizontal mirror) and then in the *y*-axis (vertical mirror).

   (b) What are the coordinates of the images of point A and B when reflected in the *x*-axis?

   ....

   (c) What are the coordinates of the images of point A and B when reflected in the *y*-axis?

   ....

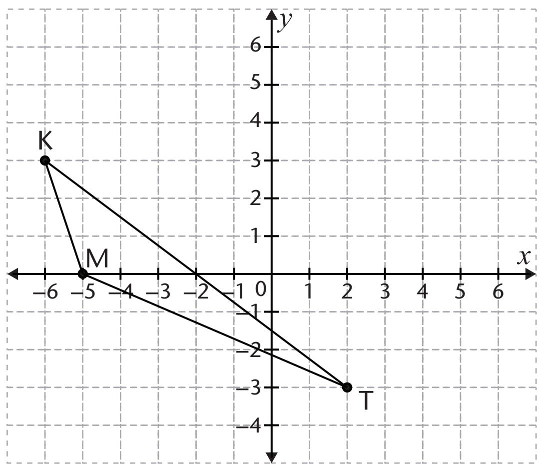
(d) Compare the coordinates of points A and B with the coordinates of their images. What do you notice?

....

....

....

2. The points K, M and T are plotted on the coordinate system.



(a) Write down the coordinates of points K, M and T.

....

(b) Reflect each point in the *x*-axis and write down the coordinates of K', M' and T'.

....

(c) Reflect points K, M and T in the *y*-axis and write down the coordinates of K'', M'' and T''.

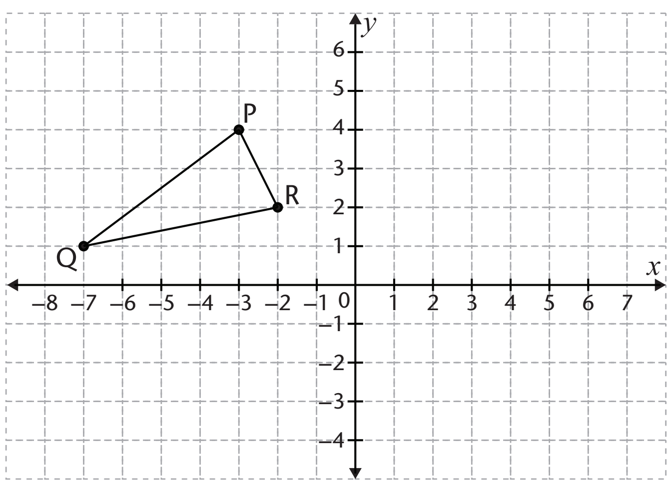
....

(d) Join points K, M and T to form a triangle. Do the same with points K', M' and T', and with points K'', M'' and T''.

(e) Are all three triangles congruent? ....

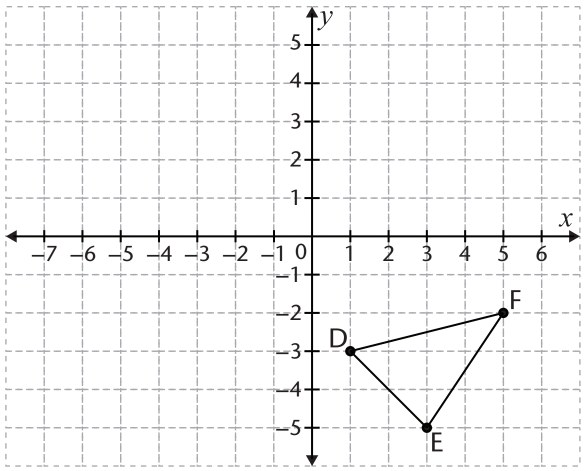
REFLECTING TRIANGLES IN THE *x*-AXIS OR IN THE *y*-AXIS

When you reflect a triangle, first reflect the vertices of the triangle and then join the reflected points.



1. (a) Reflect ΔPQR in the *x*-axis.
   (b) Reflect ΔPQR in the *y*-axis.

2. (a) Reflect ΔDEF in the *x*-axis.
   (b) Reflect ΔDEF in the *y*-axis.



3. The coordinates of the vertices of three triangles are given in the tables below. For each vertex, write down the coordinates of its reflection in the *x*-axis or in the *y*-axis as required.

(a)

Vertices of triangle	Reflection in the *x*-axis
K(–4; 5)	
L(2; –5)	
M(–5; –3)	

(b)

Vertices of triangle	Reflection in the *y*-axis
X(–1; 3)	
Y(–2; –3)	
Z(4; 1)	

(c)

Vertices of triangle	Reflection in the *y*-axis	Reflection in the *x*-axis
D(–2; 5)		
E(0; –3)		
G(2; 0)		

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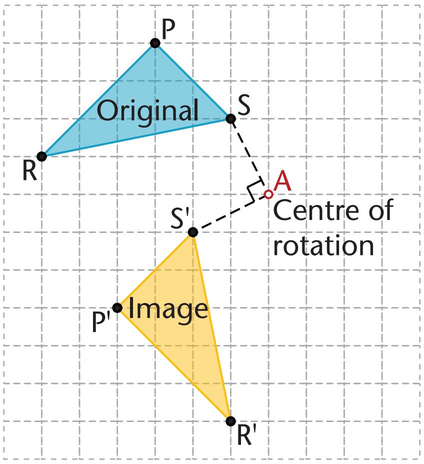
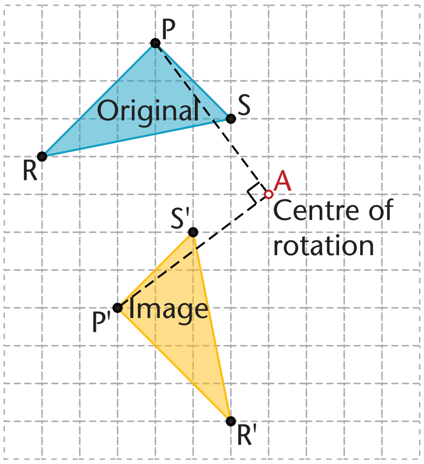
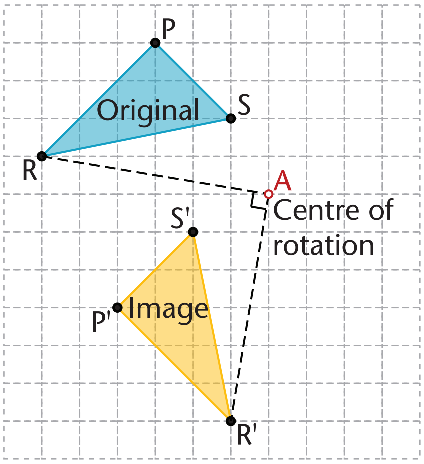
12.4 Rotation on the coordinate system

The distance from the centre of rotation to any point on the original image is equal to the distance from the centre of rotation to its corresponding point on the image. In the diagrams below: SA = S'A, PA = P'A and RA = R'A.

The angle that is formed between the line connecting an original point (S or P or R) to the centre of rotation A and the line connecting the image point (S', P', R') to the centre of rotation is equal to the angle of rotation. In the diagrams: the triangle was rotated through 90°, so $S\hat{A}S' = 90°$, $P\hat{A}P' = 90°$ and $R\hat{A}R' = 90°$.







On the coordinate system, the centre of rotation can be any point. This year, you will focus on rotations about the point (0; 0), which is called the **origin**. A point, line segment or figure can be rotated clockwise or anticlockwise through any number of degrees about the centre of rotation.

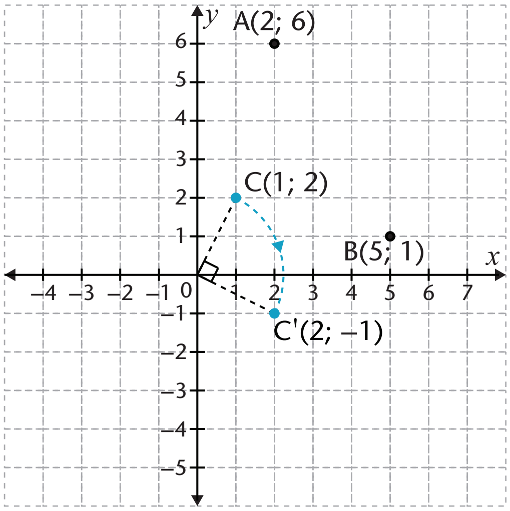
ROTATING POINTS AND FIGURES ABOUT THE ORIGIN

1. In the diagram, point C has been rotated 90° clockwise about the origin.
   (a) Rotate points A and B 90° clockwise about the origin.
   (b) Write down the coordinates of points A' and B'.

   ....

   (c) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (d) Are the triangle and its image congruent?

   ....



(e) Compare the coordinates of points A, B and C with the coordinates of their images. What do you notice?

....

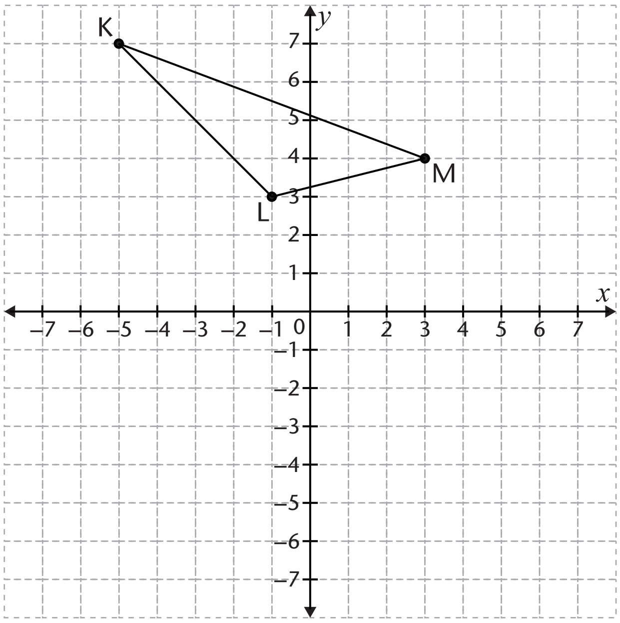
....

....

2. (a) Write down the coordinates of points K, L and M.

....

....



(b) Rotate points K, L and M 90° anticlockwise about the origin.

(c) Write down the coordinates of the image points.

....

....

(d) Rotate points K, L and M 180° about the origin.

(e) Write down the coordinates of K'', L'' and M''.

....

(f) Can you explain why there was no need to say "clockwise" or "anticlockwise" in question (d)?

....

....

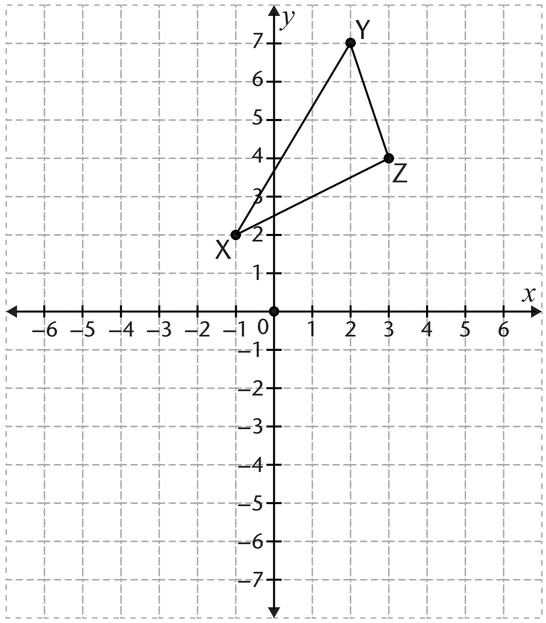
....

....

....

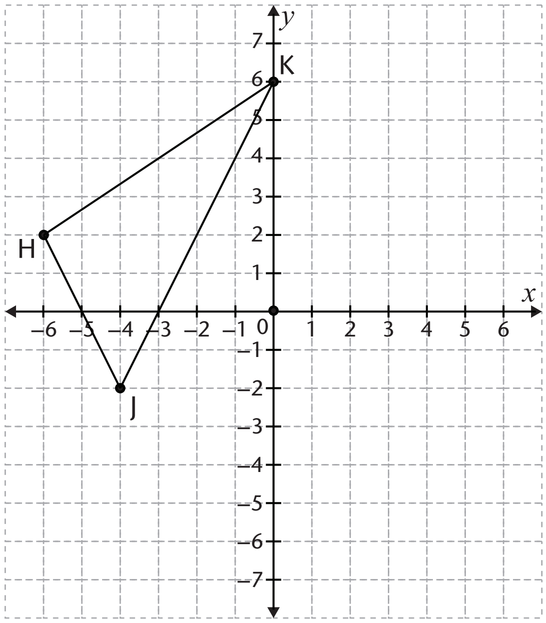
3. Rotate the following triangles and write down the coordinates of the vertices of each triangle after the required rotation.

(a) 180° about the origin



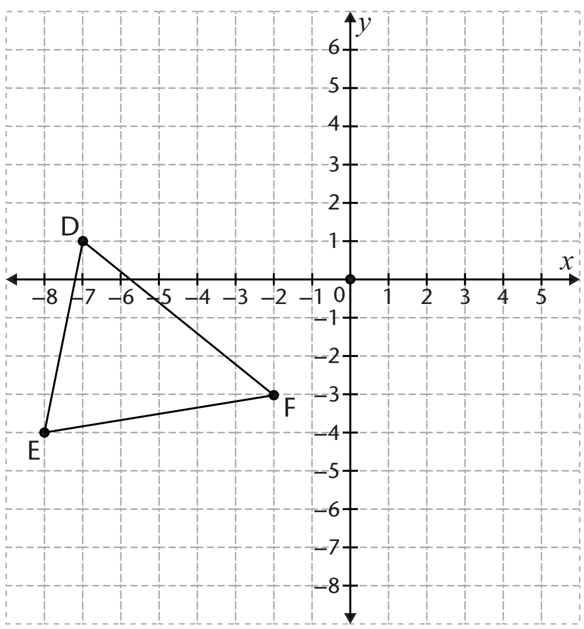
....

(b) 90° clockwise about the origin



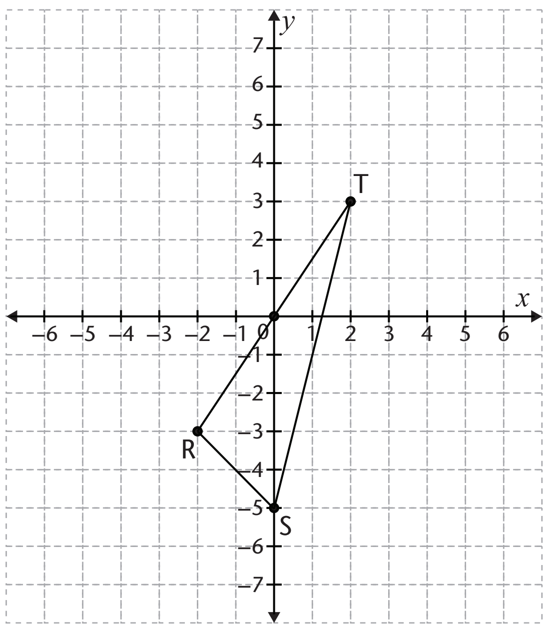
....

(c) 90° anticlockwise about the origin



....

(d) 180° about the origin



....

4. Write down the coordinates of each image point after these transformations.
   (a) Rotation 180° about the origin: K(–1; 0); C(1; 1); N(3; –2)

   ....

   (b) Rotation 90° clockwise about the origin: L(1; 3); Z(5; 5); F(4; 2)

   ....

   (c) Rotation 90° anticlockwise about the origin: S(1; –4); W(1; 0); J(3; –4)

   ....

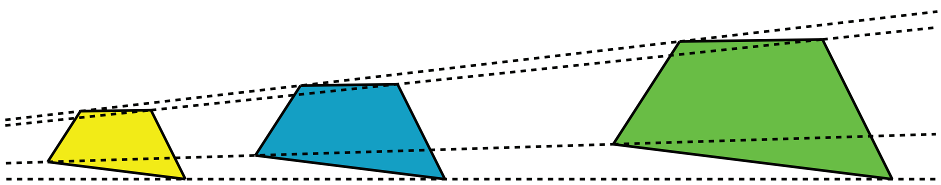
   (d) Rotation 180° about the origin: V(–5; –3); A(–3; 1); G(0; –3)

   ....

12.5 Enlargements and reductions

CALCULATE AND USE SCALE FACTORS

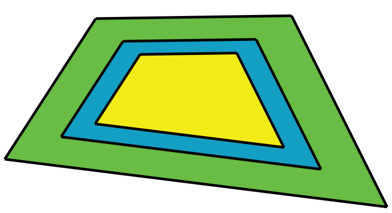
A figure may be made bigger or smaller without changing its shape.



*The yellow figure is a **reduction** of the blue quadrilateral*

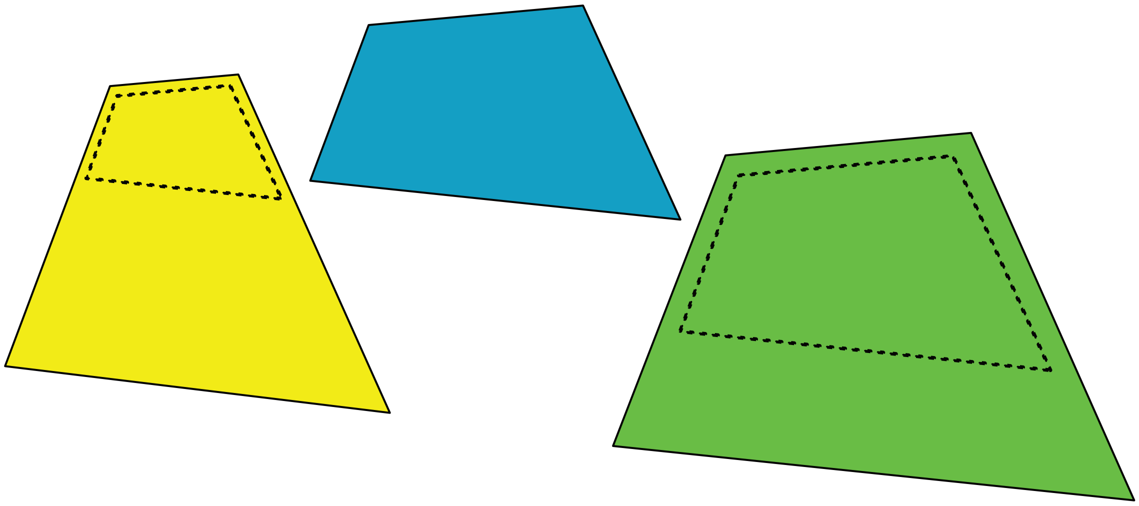
*The green figure is an **enlargement** of the blue quadrilateral*

A figure is only called an enlargement or reduction of another figure if the two figures have **the same shape**. The shapes can only be the same if all the corresponding angles are equal.



Maths2_Gr8_LB_Book.indb 188 2014/09/04 10:39:37 AM

Even if the angles are equal, two figures may have different shapes. When the corresponding angles are equal, one figure is not necessarily an enlargement or reduction of the other.



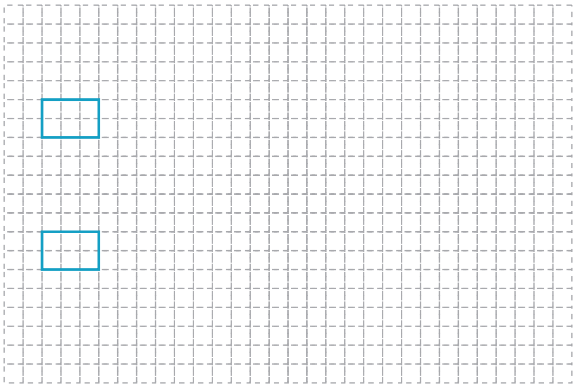
Although the angles are equal, the yellow and green figures above are *not* enlargements of the blue figure.

When a figure with straight sides is enlarged or reduced, the lengths of the sides are increased or decreased.

To find the lengths of the sides of the new figure, the lengths of the sides of the original figure are all multiplied by the same number. This number is called the **scale factor** of the enlargement or reduction.

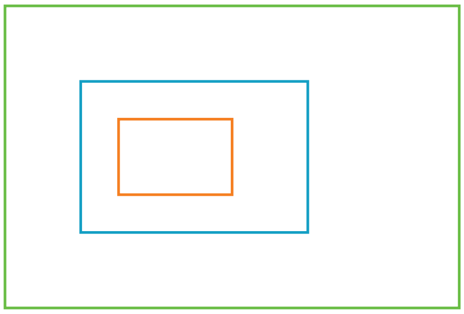
The scale factor for an **enlargement** is bigger than 1. The scale factor for a **reduction** is smaller than 1.

1. Draw a bigger rectangle ABCD on the grid below, with each side 5 times as long as the blue rectangle. Also draw another bigger rectangle PQRS, with each side 5 units longer than the blue rectangle.

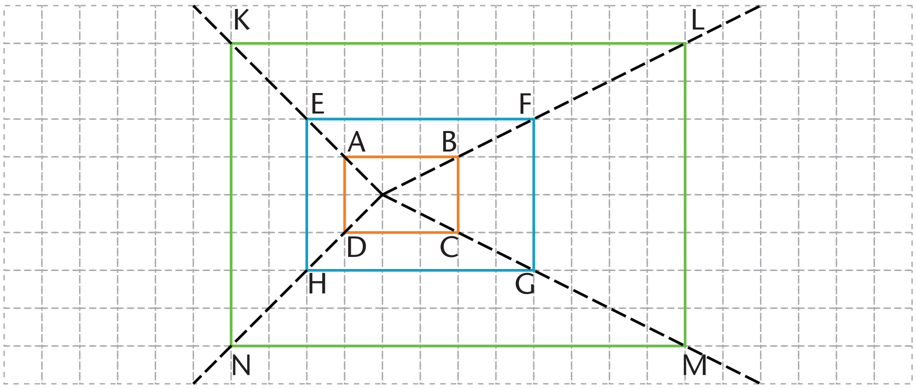


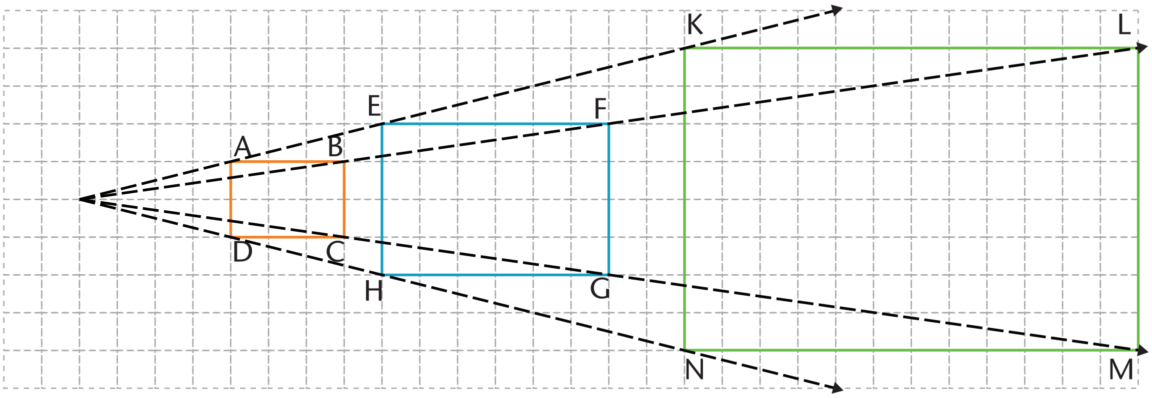
One figure is only called an enlargement or reduction of another figure if the **corresponding angles are equal** and **the ratio between the lengths of the corresponding sides is the same**, for all pairs of corresponding angles and sides in the two figures. This is demonstrated below.

The green rectangle on the right is an enlargement of the blue rectangle. The orange rectangle is a reduction of the blue rectangle.



In the two diagrams below, the same rectangles are shown on grids so that it is easy to compare the lengths of the corresponding sides and calculate the ratio between the lengths of the sides.





KLMN is an enlargement of EFGH.

Note that $\frac{LM}{FG} = 8:4 = 2$, $\frac{MN}{GH} = 12:6 = 2$, $\frac{NK}{HE} = 8:4 = 2$ and $\frac{KL}{EF} = 12:6 = 2$.

The ratio between the lengths of corresponding sides is 2, for all four pairs of corresponding sides.

We say: The **scale factor** of the enlargement from EFGH to KLMN is 2.

To avoid confusion, mathematicians normally state the dimensions of the image first when forming ratios.

ABCD is a reduction of EFGH.

Note that $\frac{BC}{FG} = 2:4 = \frac{1}{2}$, $\frac{CD}{GH} = 3:6 = \frac{1}{2}$, $\frac{DA}{HE} = 2:4 = \frac{1}{2}$ and $\frac{AB}{EF} = 3:6 = \frac{1}{2}$.

The ratio between the lengths of corresponding sides is $\frac{1}{2}$, for all four pairs of corresponding sides. The scale factor of the reduction from EFGH to ABCD is $\frac{1}{2}$.

2. (a) What is the scale factor of the enlargement from ABCD to KLMN?

....

(b) What is the scale factor of the reduction from KLMN to EFGH?

....

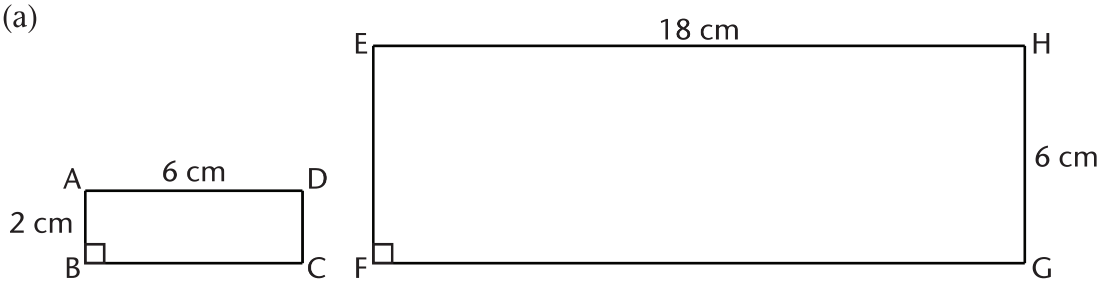
3. A rectangular shape on a photograph is 3 mm wide and 4 mm long. The photograph is enlarged with a scale factor of 5. What is the width and length of the rectangular shape on the enlarged photograph?

....

We work out the scale factor by calculating the ratios of the lengths of corresponding sides of the two figures. If the ratios are equal, we say that the corresponding sides are **in proportion**. This means that the second figure (the image) is a reduction or an enlargement of the first figure (the original).

4. Determine whether the second figure in each of the following pairs is an enlargement, a reduction, or neither of the two. Also work out the perimeters of both figures.

(a)



....

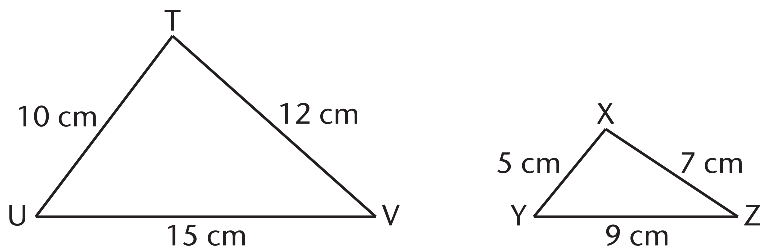
....

....

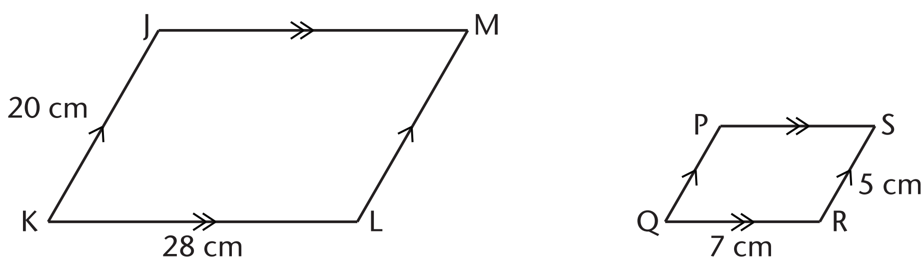
....

....

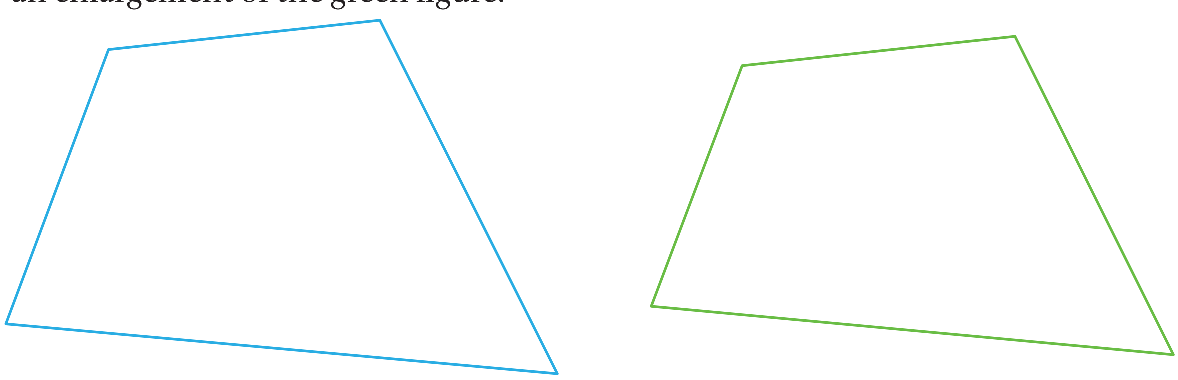
(b)



(c)

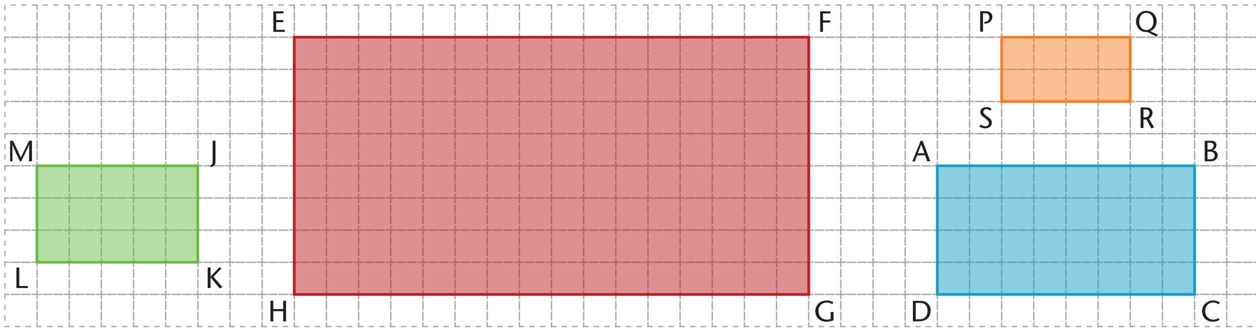


5. Take measurements and do calculations to establish whether the blue figure below is an enlargement of the green figure.



EFFECT OF ENLARGEMENTS OR REDUCTIONS ON PERIMETER AND AREA

Consider the rectangles below.



1. (a) Do you think EFGH is an enlargement of MJKL? ....

   (b) Do you think PQRS is a reduction of EFGH? ....

   (c) Do you think EFGH is an enlargement of ABCD? ....

2. (a) Calculate $\frac{EF}{MJ}$, $\frac{FG}{JK}$, $\frac{GH}{KL}$ and $\frac{HE}{LM}$.

   ....

   (b) Is rectangle EFGH an enlargement of rectangle MJKL? ....

   (c) If EFGH is an enlargement of MJKL, what is the scale factor? ....

3. (a) Calculate $\frac{PQ}{EF}$, $\frac{QR}{FG}$, $\frac{RS}{GH}$ and $\frac{SP}{HE}$.

   ....

   (b) Is rectangle PQRS a reduction of rectangle EFGH? ....

   (c) If PQRS is a reduction of EFGH, what is the scale factor? ....

4. (a) Calculate $\frac{EF}{AB}$, $\frac{FG}{BC}$, $\frac{GH}{CD}$ and $\frac{HE}{DA}$.

   ....

   (b) Is rectangle EFGH an enlargement of rectangle ABCD? ....

   (c) If EFGH is an enlargement of ABCD, what is the scale factor? ....

5. Do you agree or disagree with the following statements?

   (a) Perimeter of enlargement/reduction = perimeter of original × scale factor ......

   (b) Area of enlargement/reduction = area of original × (scale factor)^2 ....

Maths2_Gr8_LB_Book.indb 193 2014/09/04 10:39:42 AM

CALCULATING PERIMETERS AND AREAS OF ENLARGED OR REDUCED FIGURES

1. The perimeter of rectangle DEFG = 20 cm and its area = 16 cm^2. Find the perimeter and area of the enlarged rectangle D'E'F'G' if the scale factor is 3.

   ....

   ....

2. The perimeter of ΔJKL = 120 cm and its area = 600 cm^2. Determine the perimeter and area of the reduced ΔJ'K'L' if the scale factor is 0,5.

   ....

   ....

3. The perimeter of quadrilateral PQRS = 30 mm and its area is 50 mm^2. Find the perimeter and area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.

   ....

   ....

4. The perimeter of ΔSTU = 51 cm and its area is 12 cm^2. Calculate the perimeter and area of ΔS'T'U' if the scale factor is $\frac{1}{3}$.

   ....

   ....

5. The perimeter of a square = 48 m.
   (a) Write down the perimeter of the square if the length of each side is doubled.

   ....

   (b) Will the area of the enlarged square be twice or four times that of the original square? ....

6. The perimeter of ΔDEF = 7 cm and ΔD'E'F' = 21 cm. What is the scale factor of enlargement? How many times larger is the area of ΔD'E'F' than the area of ΔDEF?

   ....

   ....

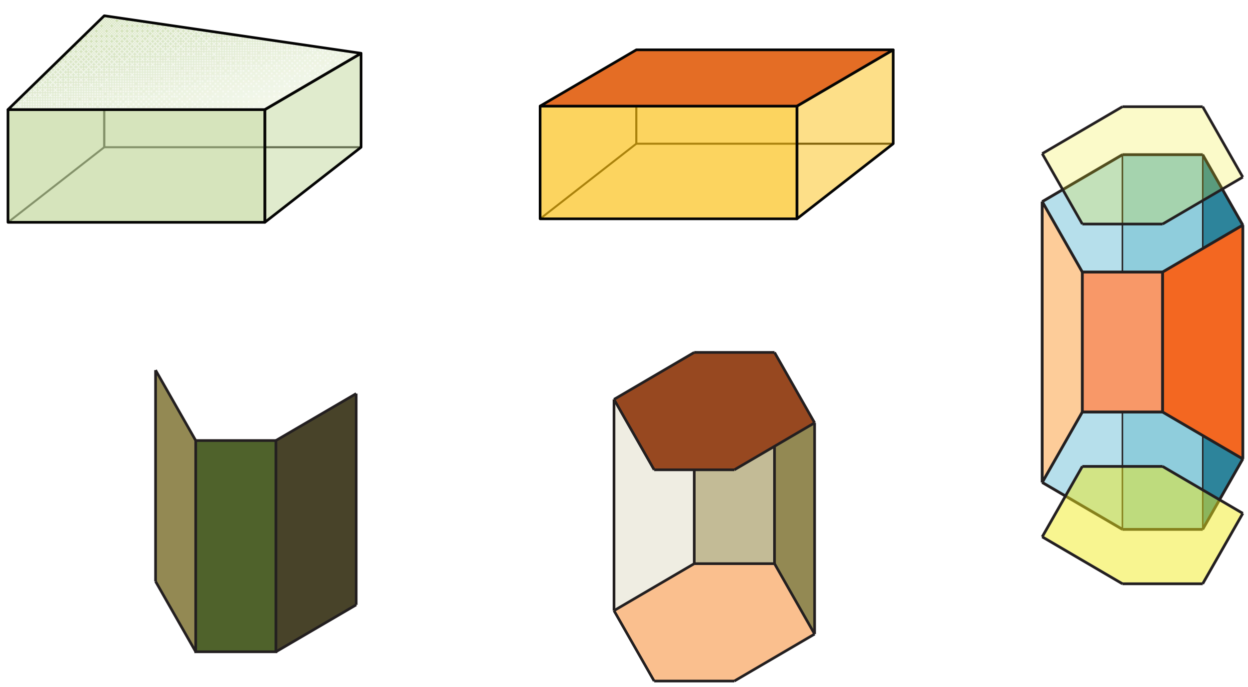
7. The perimeter of quadrilateral ADFS = 26 cm and the perimeter of quadrilateral A'D'F'S' = 13 cm. How many times larger is the area of quadrilateral A'D'F'S' than the area of quadrilateral ADFS?

   ....

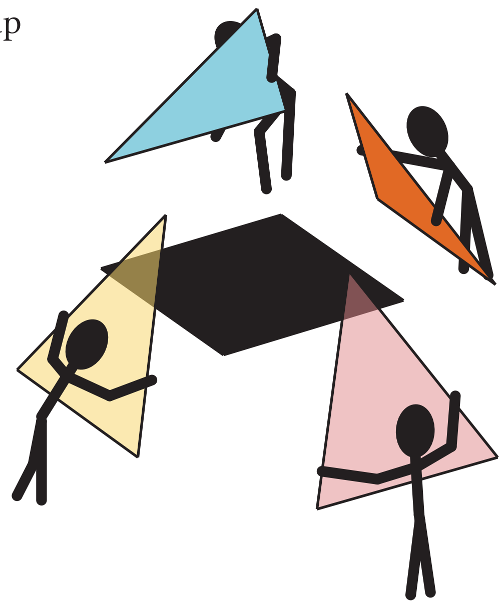
   ....

Chapter 13
Geometry of 3D objects

In this chapter, you will revise what you should already know about different types of 3D objects and how they can be described in terms of the number and shape of their faces, number of vertices and number of edges. You will draw accurate nets and construct models of prisms and pyramids. You will learn about a surprising relationship between the numbers of vertices, edges and faces of different polyhedra. You will also investigate the so-called "Platonic solids".

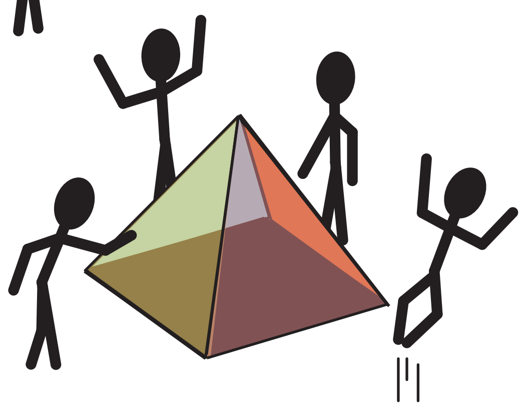


The four men want to put the four glass sheets up around the black base to build a pyramid.

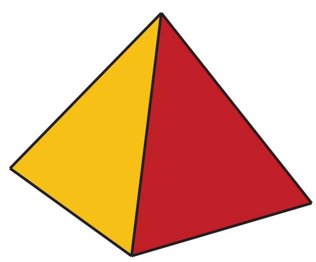


The yellow and pink glass sheets are in place now.

The job is done, the pyramid stands!
Do you want to be inside it?
They now decide to paint the four glass sides so that one cannot see inside.



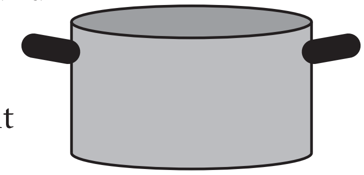
Now you cannot see inside, and you cannot tell that this diagram represents a 3D object.



13 Geometry of 3D objects

13.1 Revision: 3D objects

THINK OF SPACE WHILE YOU LOOK AT PICTURES AND DRAWINGS



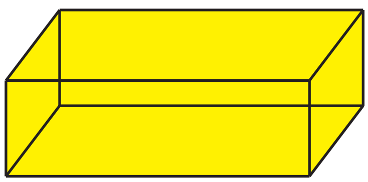
Most objects we see around us, like fruit, animals, trees, people and motor cars, have curved or round surfaces. Some objects, like a saucepan or other cooking vessel, have both round and flat surfaces. The circular bottom of a saucepan must be flat so that it makes good contact with the stove plate.

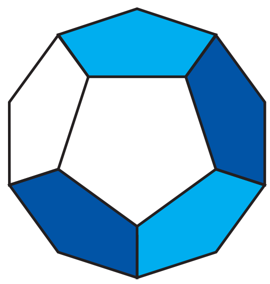
1. (a) Should the top of a table or desk be a flat or curved surface? ....

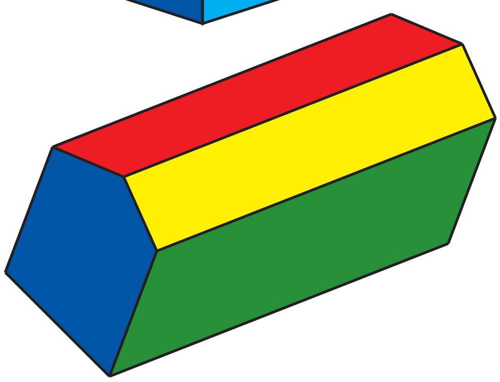
   (b) We eat with knives, forks and spoons. Which of these objects normally have curved surfaces?

   ....

This chapter is about objects that only have flat surfaces, like those shown below.







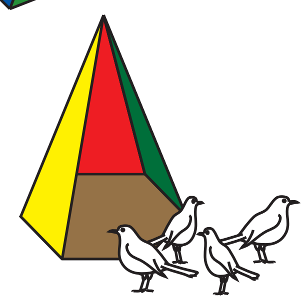
The front, right and top faces in the above drawing are made of clear plastic so that you can see the faces behind them.

Note this strange box with different colours on its different faces.

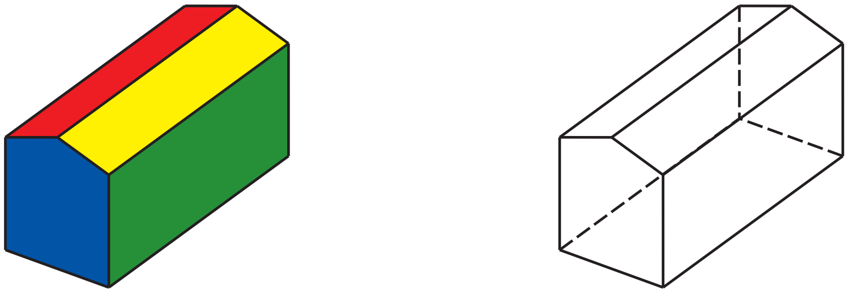
2. Do you think there is enough space for all the birds in the tent shown on the right-hand side?

   ....

   ....



3. The unusual box, shown on the previous page, with flat faces (surfaces) only is shown again below. In the drawing of the same box on the right, dotted lines are used to indicate edges and surfaces that are hidden in the coloured drawing.



(a) How many faces (different flat surfaces) does this object have altogether? ....

(b) How many faces cannot be seen in the coloured drawing on the left? ....

(c) How many of the faces are rectangles? ....

(d) How many of the faces are pentagons? ....

A 3D object with **flat faces (surfaces) only** is called a **polyhedron** (plural: **polyhedra**).

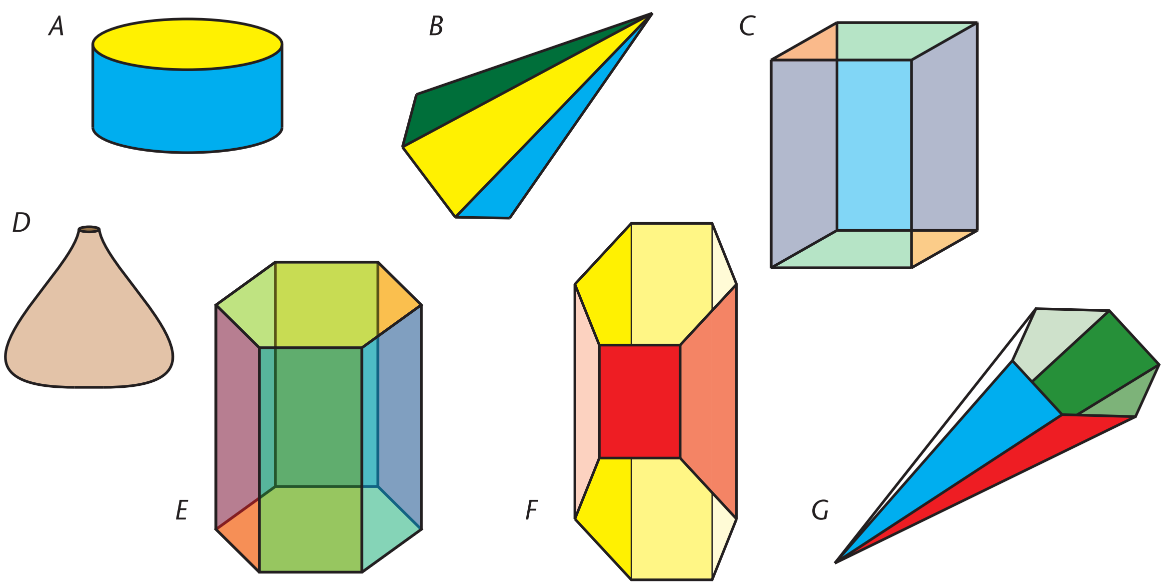
A straight **edge** is formed where two flat surfaces meet. The point where two or more edges meet is called a **vertex** (plural: **vertices**).

The word **polyhedron** means 'many-seated' and describes the shape of such an object with many flat faces.

4. (a) How many edges does the coloured polyhedron in question 3 have? ....

(b) How many vertices does it have? ....

5. Which of the objects below are polyhedra? ....

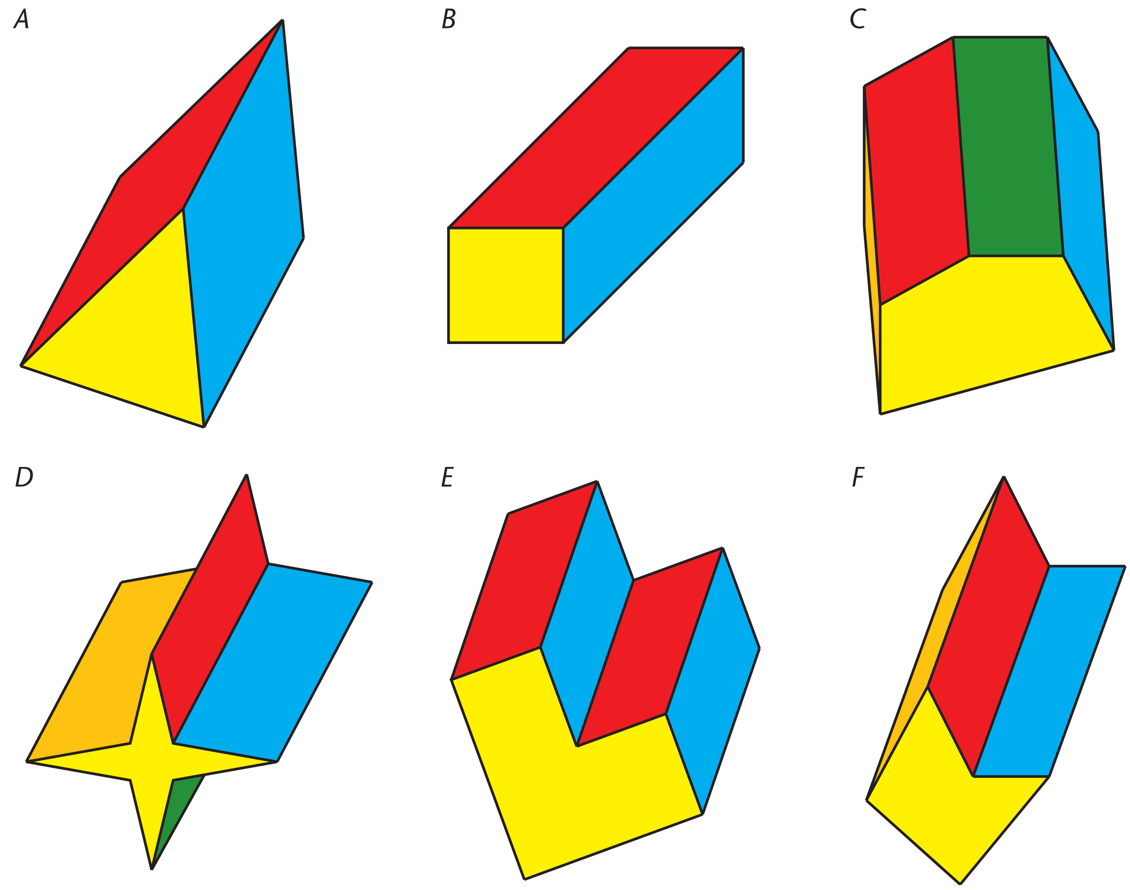


TWO SPECIAL TYPES OF POLYHEDRA

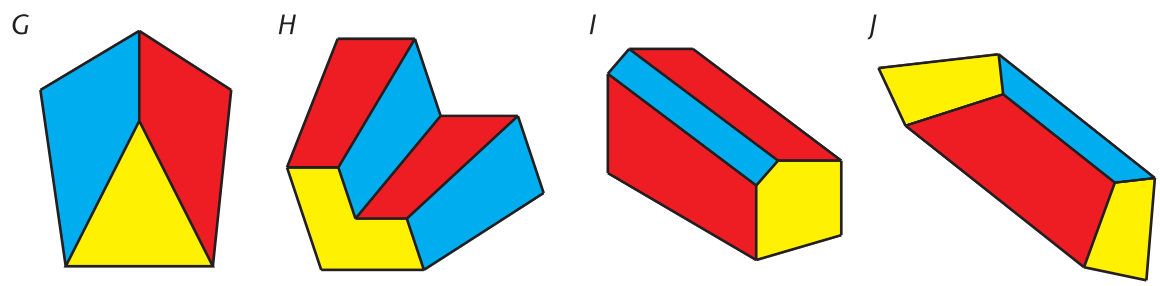
Polyhedra like C and E at the bottom of the previous page are called **prisms**. Polyhedra like B and G are called **pyramids**.

1. Describe the differences between prisms and pyramids.

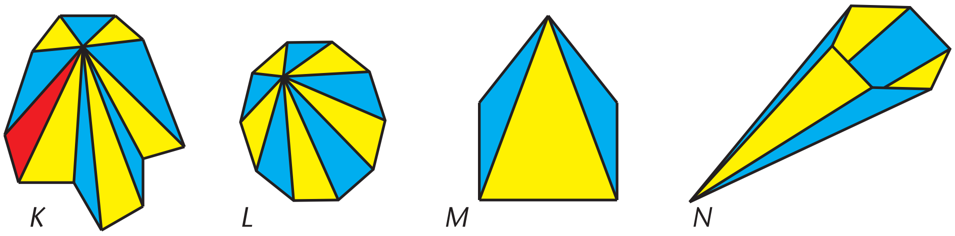
Here are some more pictures of **prisms**.



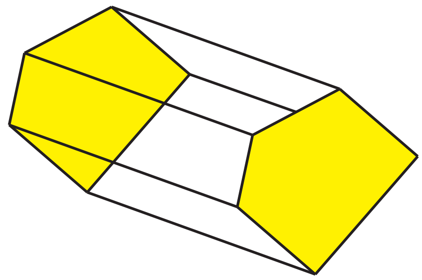
The objects shown by the four pictures below are polyhedra but they are *not* prisms or pyramids.



Here are some pictures of **pyramids**. More pictures of pyramids are shown at the bottom of this page and also on the next page.



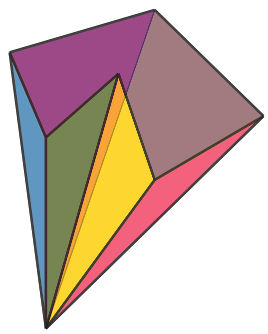
A **prism** has two identical, parallel faces (called **bases**) that are connected by parallelograms (called **lateral faces**). In the case of right prisms, the lateral faces are perpendicular to the bases and the lateral faces are rectangles.



A prism with pentagonal bases like this one is called a **pentagonal prism** because the base is a pentagon.

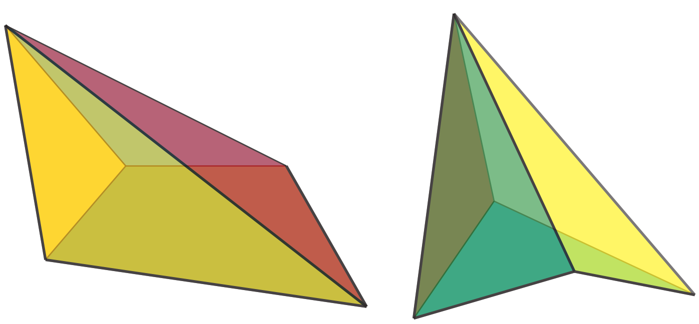
2. (a) Which pictures on the previous page also show pentagonal prisms? ....
   (b) Which picture on the previous page shows a hexagonal prism? ....
   (c) Which picture on the previous page shows an octagonal prism? ....

A **pyramid** has only one base. The lateral faces of a pyramid are triangles that meet at the **apex**.



The pyramid on the right is called a **hexagonal-based pyramid**.

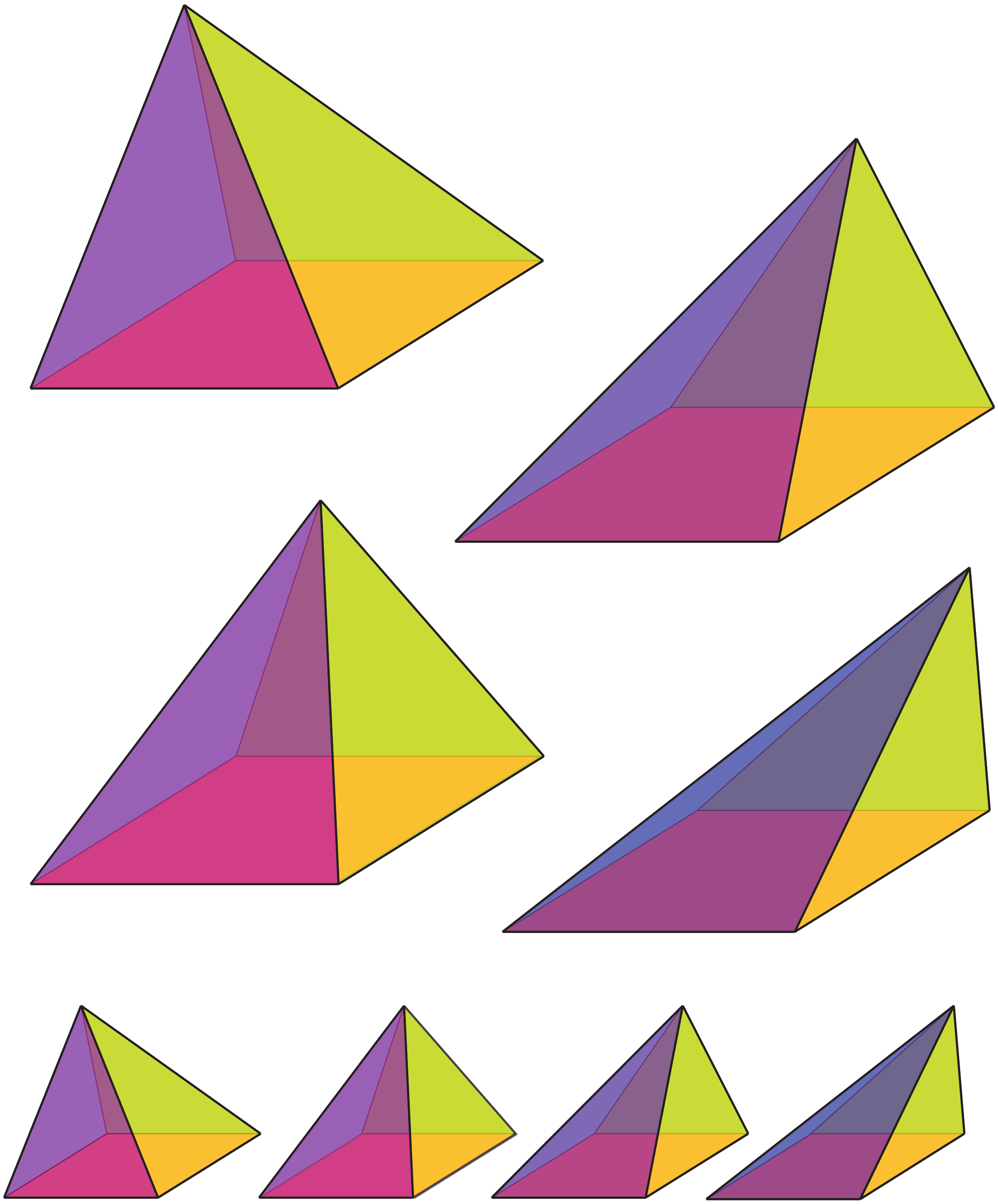
The two pyramids below have quadrilaterals as bases and are called **quadrilateral-based pyramids**.



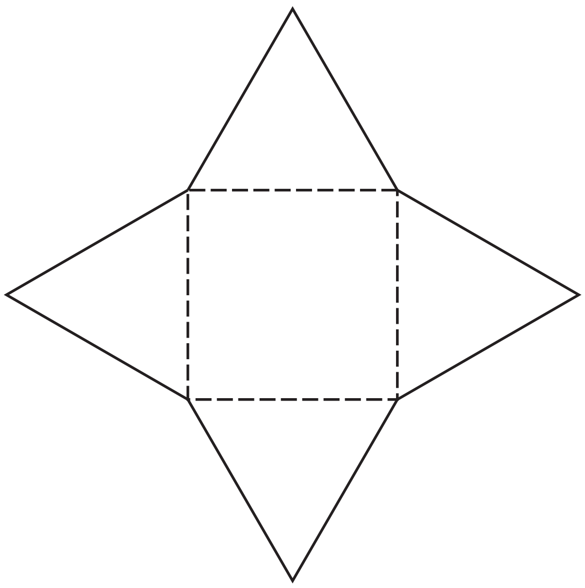
A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

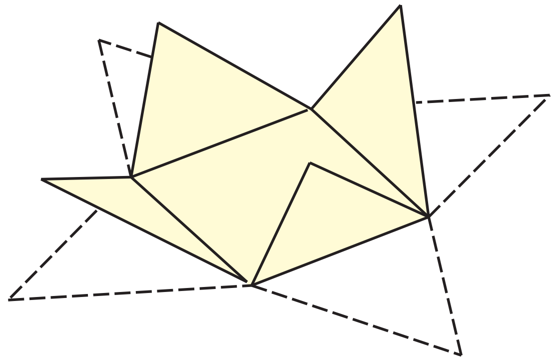
3. Which picture at the top of this page shows a hexagonal pyramid? ....

Pictures of different **square-based pyramids** are shown below.

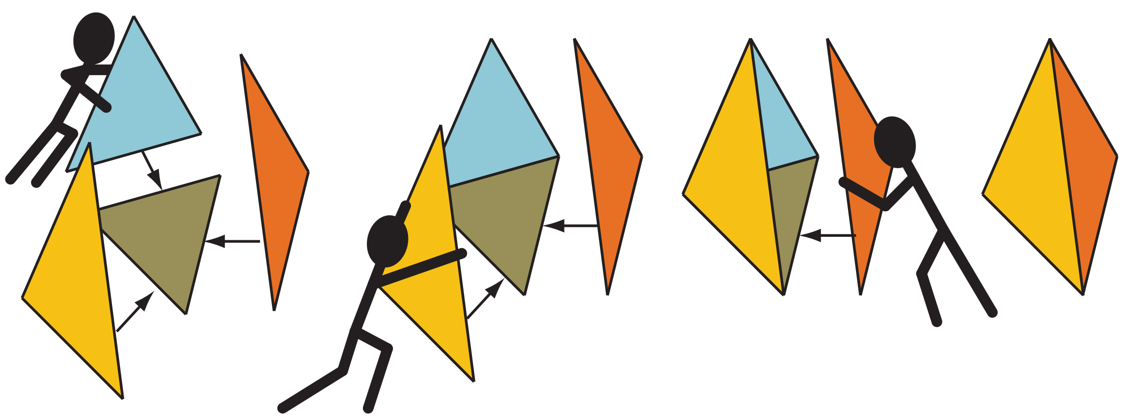


You can make a square-based pyramid by drawing and cutting out a diagram like the one on the left below, and folding the triangles up on the dotted lines as shown on the right.



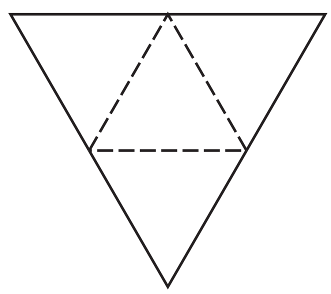


These men are building a **triangular-based** pyramid.



A triangular-based pyramid is also called a **tetrahedron**, which literally means "four-face".

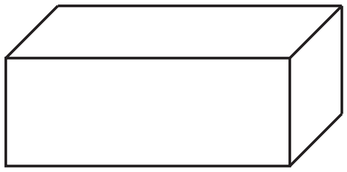
A tetrahedron with four identical faces that are equilateral triangles is called a **regular tetrahedron**.



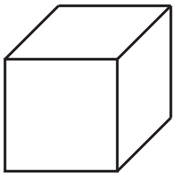
If you draw and cut out a figure like the one on the right, and fold the triangles up on the dotted lines, you can make a regular tetrahedron. A diagram like this, that can be cut out and folded to make a model of a polyhedron, is called a **net**.

A **regular polyhedron** has identical faces that are regular polygons, i.e. with all sides and angles equal.

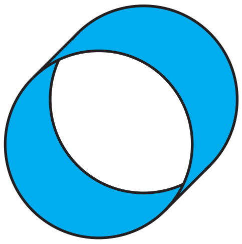
A rectangular prism is also called a **cuboid**.



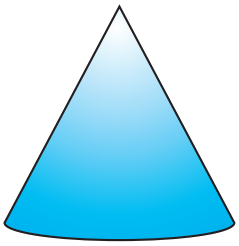
A cuboid with square faces is also called a **cube**.



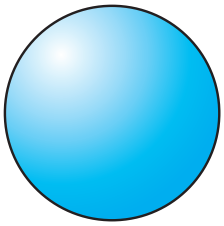
An object with two identical circular bases and one curved surface is called a **cylinder**.



A "pyramid" with a round base is called a **cone**.

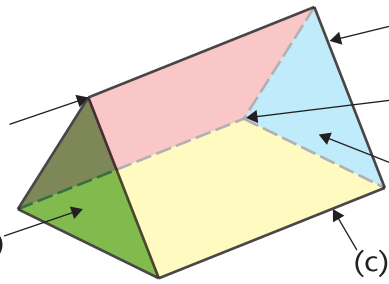


An object with the shape of a ball, in other words one curved surface with every point on its surface the same distance from its centre, is called a **sphere**.



Cylinders, cones and spheres are *not* polyhedra since they have curved surfaces. Remember, a polyhedron has faces, edges and vertices. The faces are the flat surfaces. An edge is a line along which two faces of a 3D object meet; an edge connects two vertices. A vertex is the point where the edges meet.

4. Label parts (a) to (f) on the figure below.
   (a) ....
   (b) ....
   (c) ....
   (d) ....
   (e) ....
   (f) ....

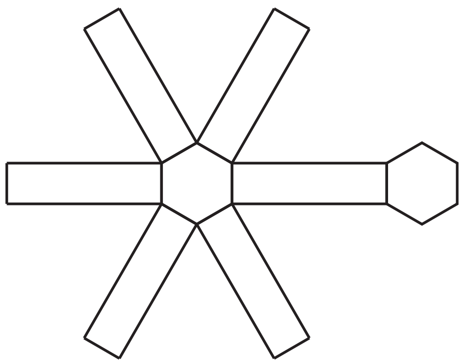


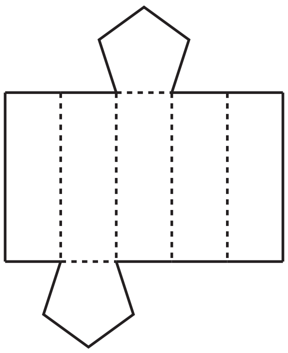
Triangular prism

5. Learners in a Grade 8 class made 3D objects from cardboard. Can you say which kind of figure the following three learners made?
   (a) Adam's object had 8 vertices and 12 edges. ....
   (b) Lea's object had 4 vertices and 4 faces. ....
   (c) Mary's object had 12 edges and 6 congruent faces. ....

6. Complete the table for prisms. Count the bases as faces too. If you find this difficult, it may help you to make quick rough sketches of nets for some prisms, like the sketches given below the table.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	5	6	9		
4	6		12		
5					
6					
8					
10					





7. Complete the table for pyramids. Count the bases as faces too.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	4	4	6		
4					
5					
6					
7					
9					

8. Consider your answers for questions 6 and 7. Is the statement below true both for prisms and pyramids?

This statement is called **Euler's formula** for polyhedra.

the number of faces + the number of vertices = 2 + the number of edges

....

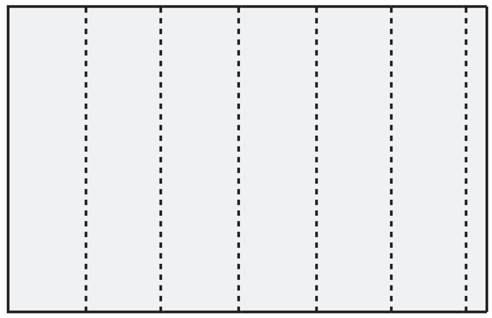
9. Is Euler's formula true for the polyhedra G, H, I and J on page 199? ....

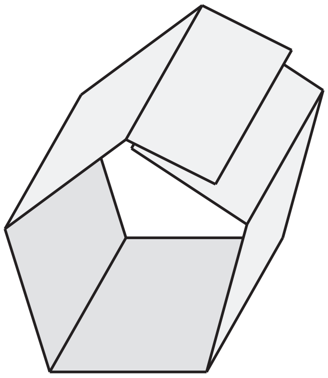
13.2 Nets and models of prisms and pyramids

A QUICK WAY TO MAKE PRISMS AND PYRAMIDS

Fold sections about two fingers wide on a sheet of A4 paper, more or less as shown by the dotted lines in the sketch on the right.

Fold the sheet into a "tube" with 5 or 6 faces along its length, as shown below.

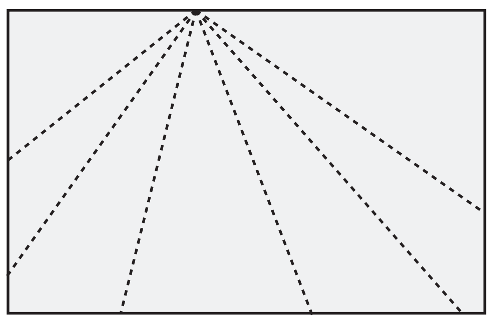


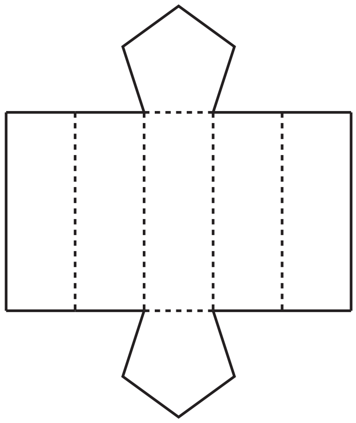


With a little extra work, you can now make a paper prism. You need to cut out two bases so that they fit well.

You can make prisms with triangular, square, rectangular, hexagonal and other shape bases in this way.

You can make a pyramid in the same way, but it is more difficult. Draw dotted lines on a sheet of A4 paper as shown in the sketch on the right. Fold the paper along the dotted lines. It is quite difficult to know where and how to cut so that the base is a flat surface.



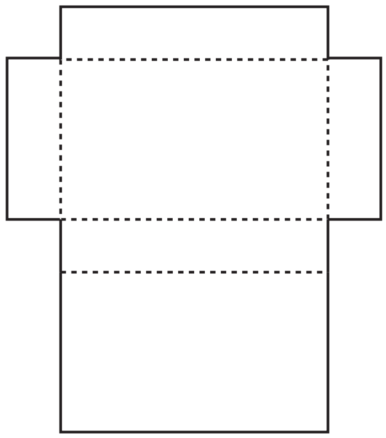


Apart from the difficulty of getting the base of the pyramid flat, the above method has the disadvantage that you have to use separate pieces of paper or other material to make one object. It would be better to make the whole object by folding one piece of paper. For example, a prism with pentagonal bases can be made by drawing, cutting out and folding a sheet of paper as shown on the left. This diagram is called a **net** of a prism with a regular pentagonal base.

NETS FOR DIFFERENT POLYHEDRA

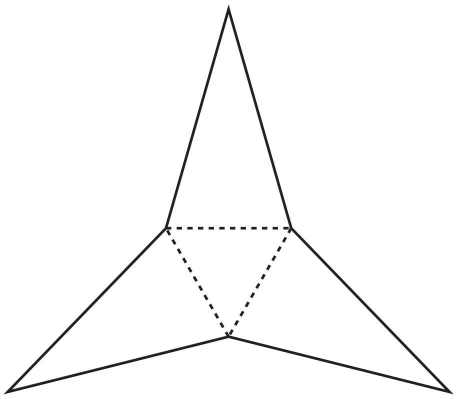
1. Name the polyhedron that can be made from each of the following nets.

(a)



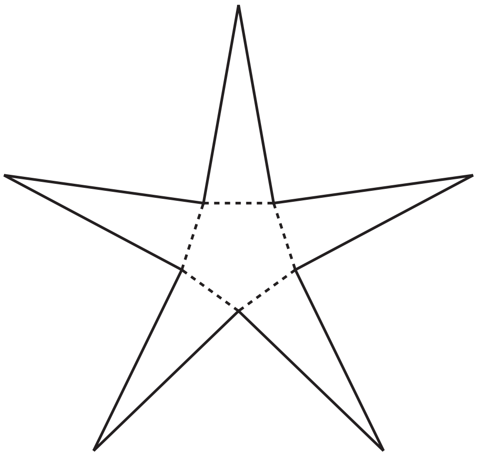
....

(b)



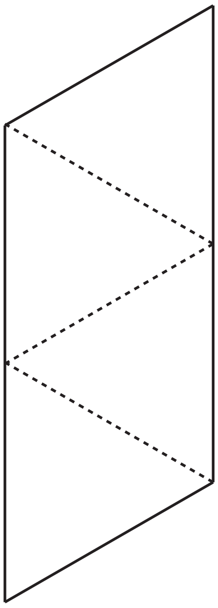
....

(c)



....

(d)

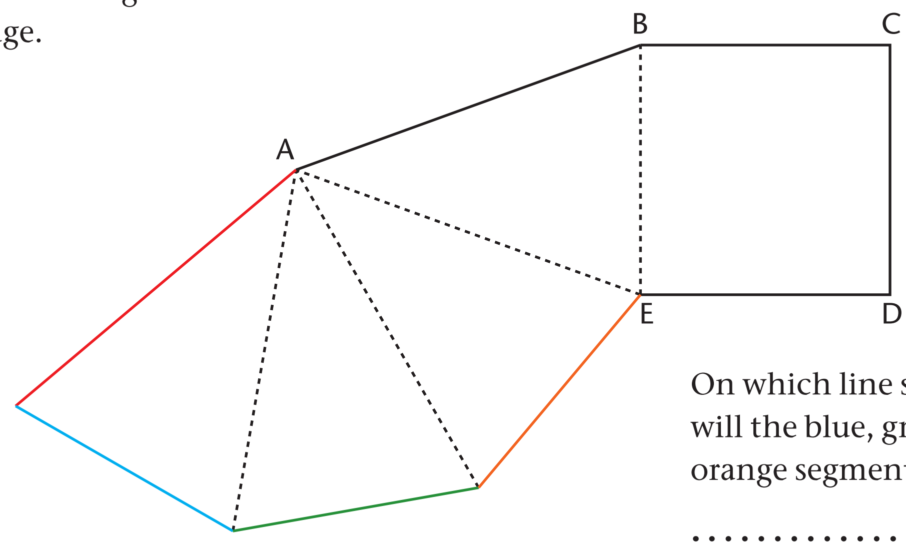


....

2. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.

....

(b) When this net is folded to make a polyhedron, the red line segment will fall on AB to form an edge.



On which line segments will the blue, green and orange segments fall?

....

....

....

3. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.

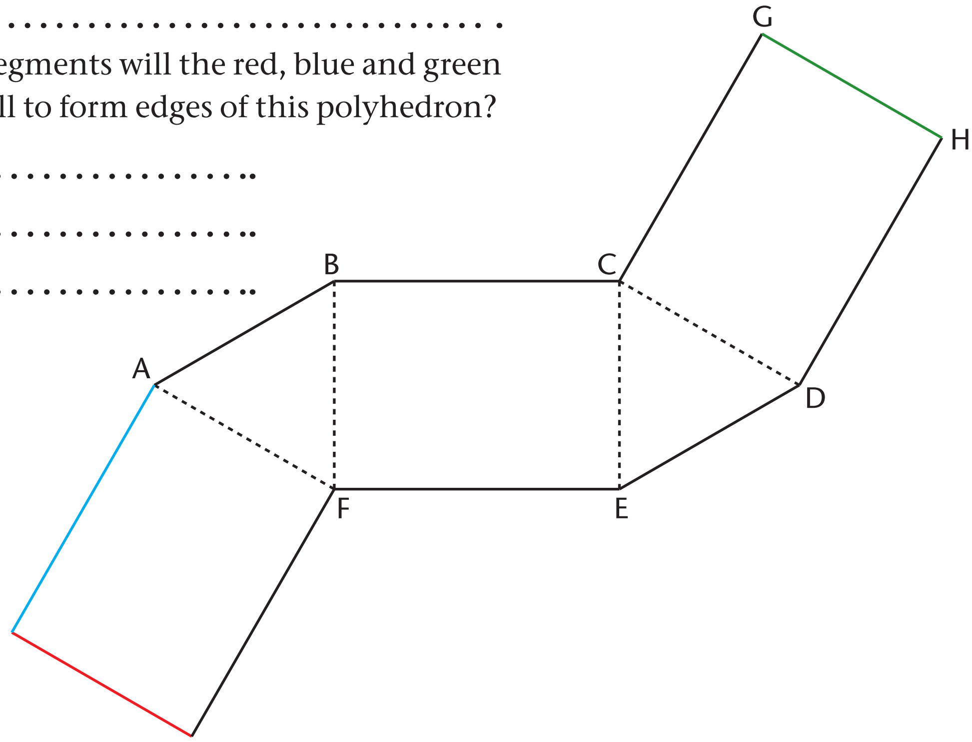
....

(b) On which segments will the red, blue and green segments fall to form edges of this polyhedron?

....

....

....



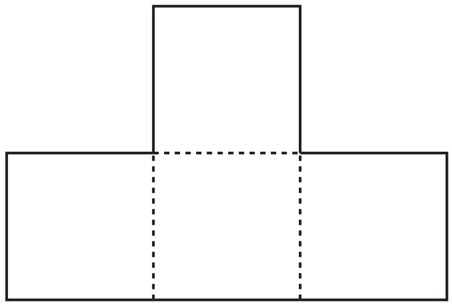
4. Some of the diagrams below and on the next page are the nets for the following objects:

a square-based pyramid	a triangular prism
a hexagonal pyramid	a hexagonal prism
a cuboid	a cube

Under each diagram, write the name of the object for which the diagram is a net. There may be more than one net for some of the objects. Write "none" if the diagram is not a net for any prism or pyramid.

A diagram is only called a **net** of an object if the cut-out diagram can be folded to form **all** the faces of the object.

(a)



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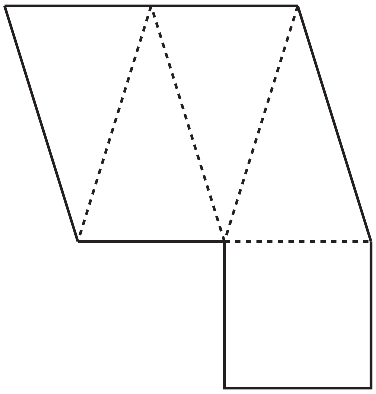
(b)



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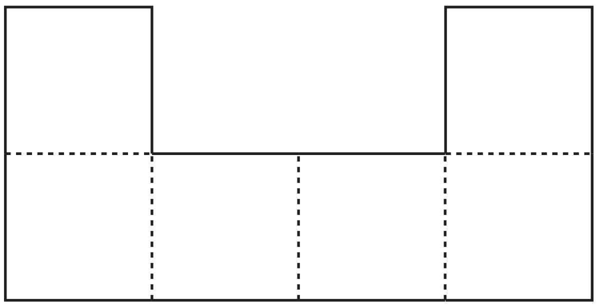
(c)



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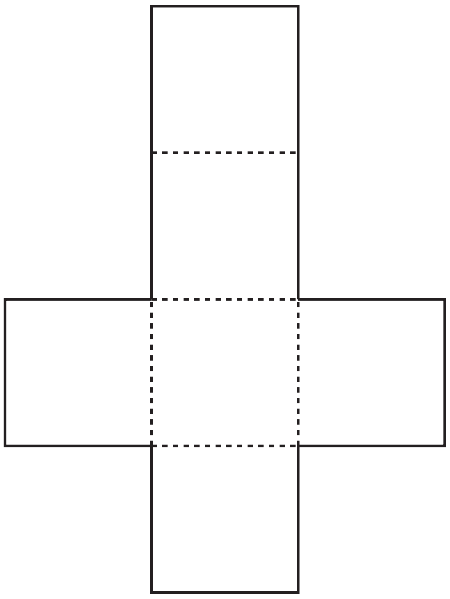
(d)



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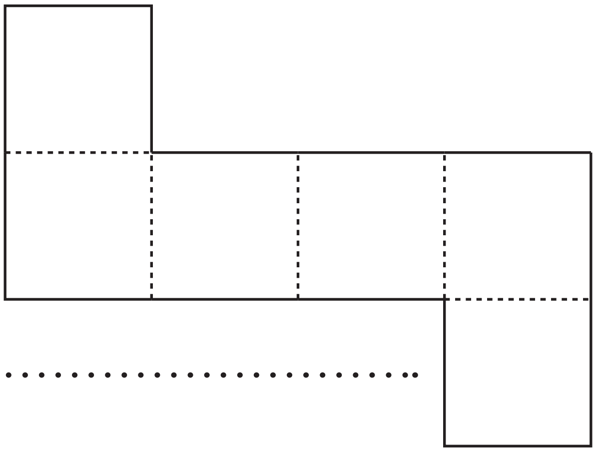
(e)



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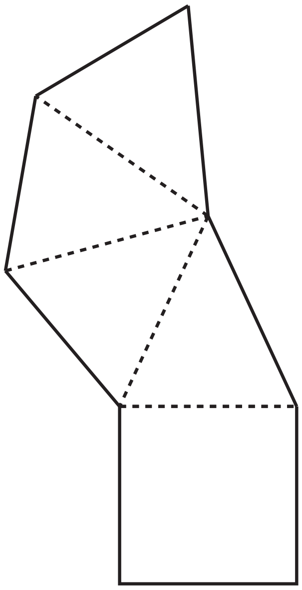
(f)



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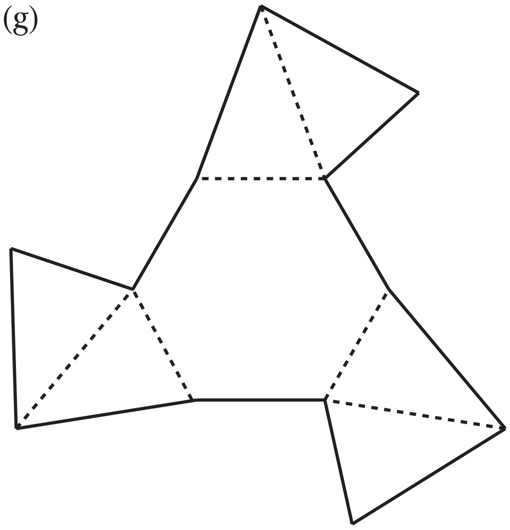
(g)



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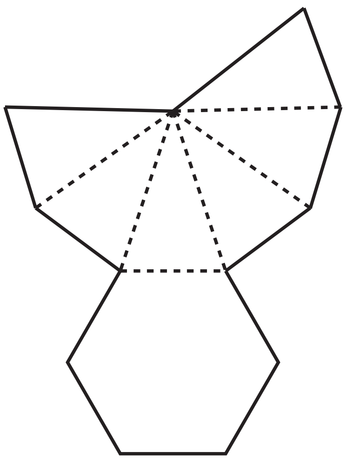
(h)



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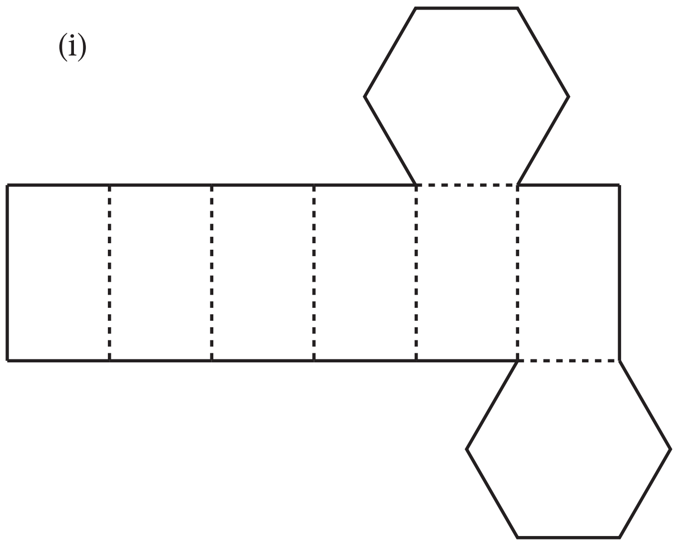
(i)



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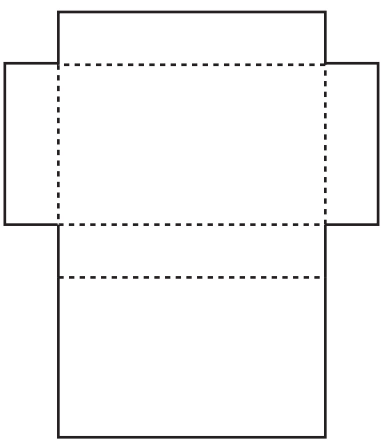
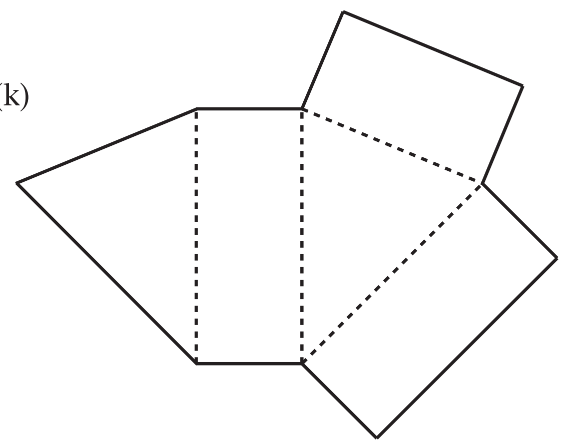
(j)



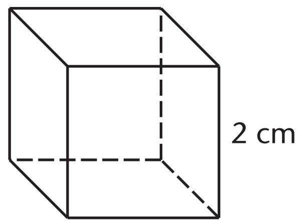
....

(k)

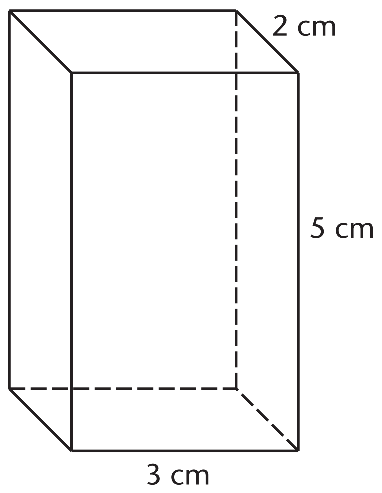


....

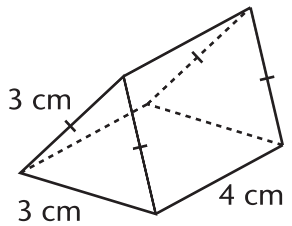
5. Draw a net for each of the following objects. Be accurate in your measurements.
   (a) Cube



   (b) Rectangular prism



   (c) Triangular prism



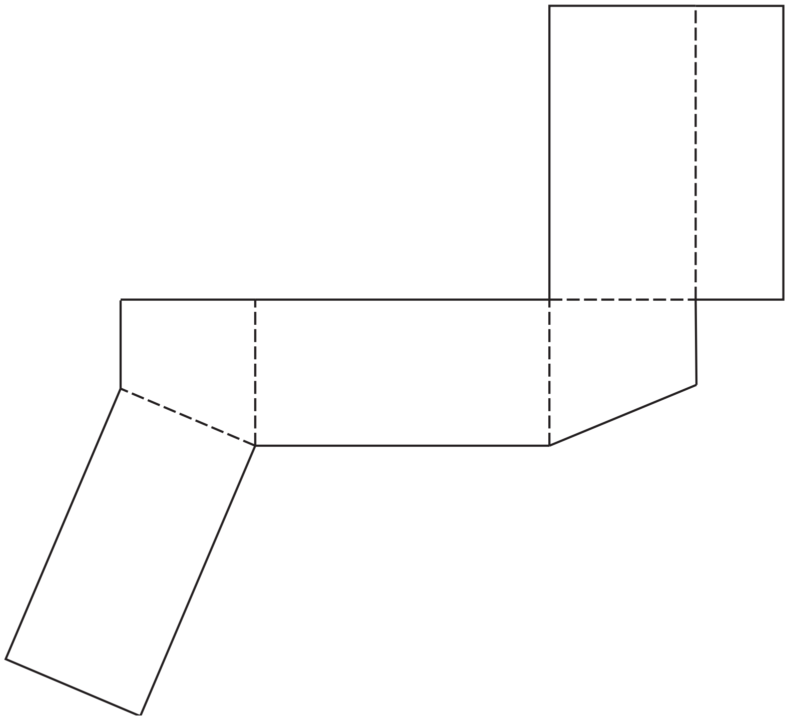
6. (a) Copy the nets in question 5 onto cardboard, but multiply the length of each side by 2. Be accurate in your constructions.
   (b) Cut out, fold and use sticky tape to paste the nets to build the 3D models.

7. The first diagram below is a net for a prism with quadrilateral bases. Which of the diagrams (a), (b), (c) and (d) are nets for the same prism, and which diagrams are not nets for the prism?

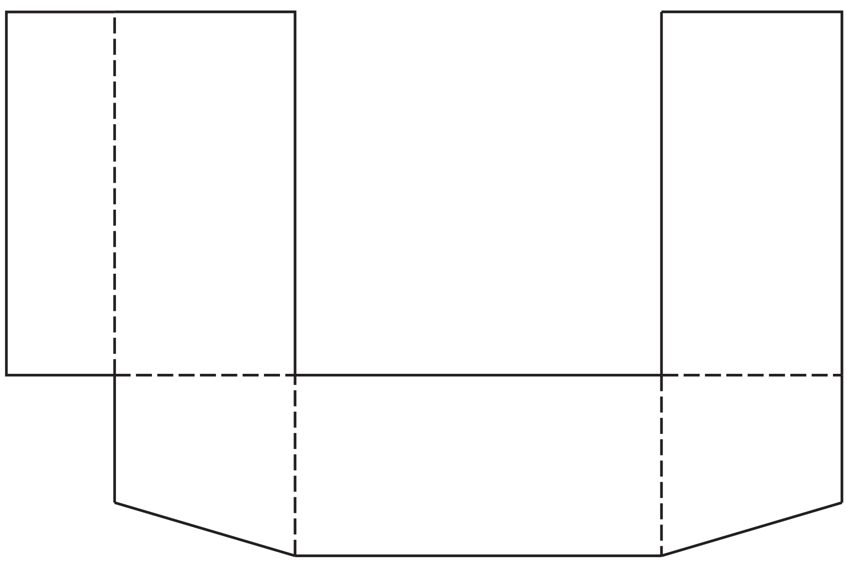
....

....

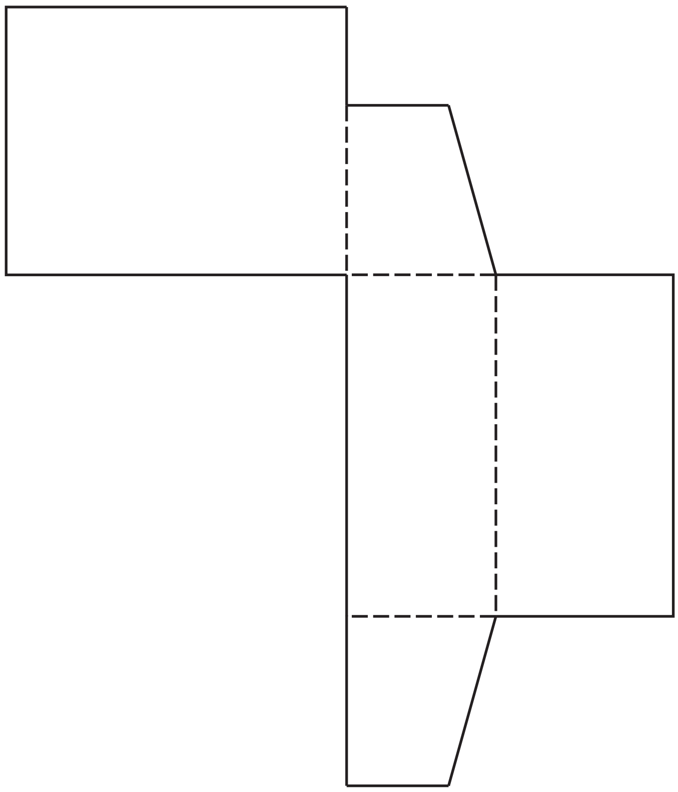
....



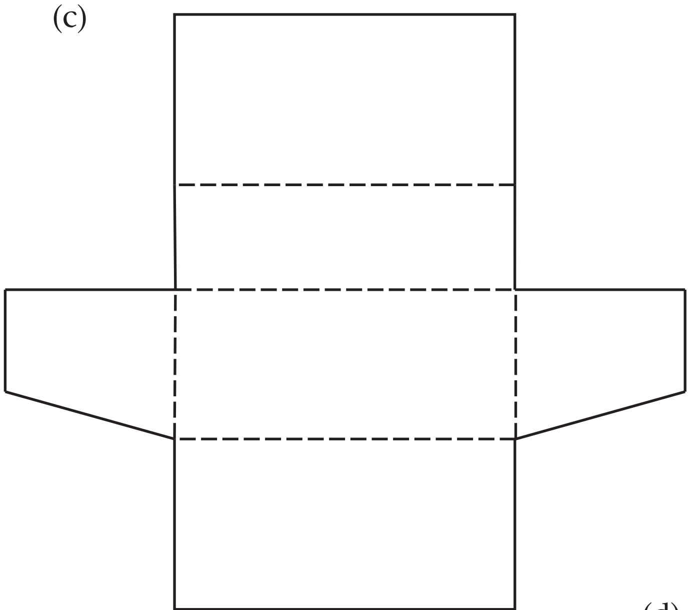
(a)



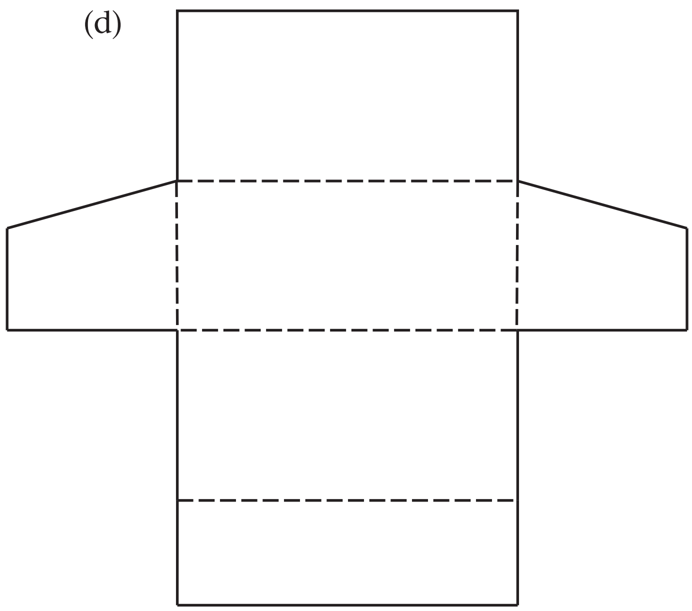
(b)



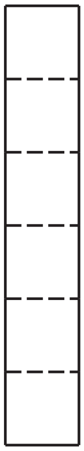
(c)



(d)



8. Which of these diagrams will work as nets for a cube?

(a)

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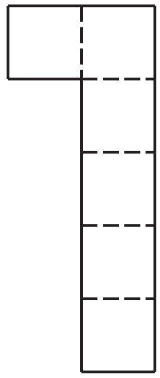
(b)

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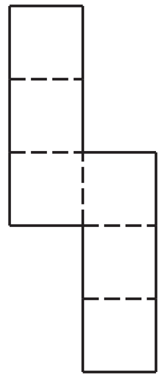
(c)

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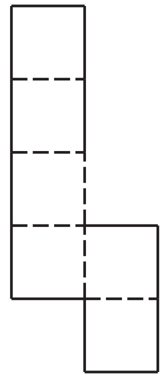
(d)

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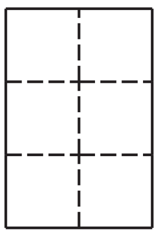
(e)

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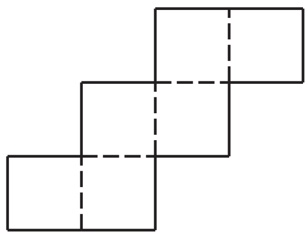
(f)

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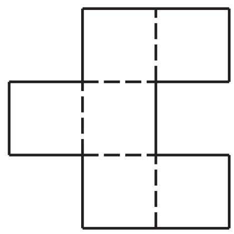
(g)

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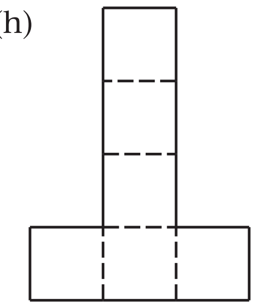
(h)

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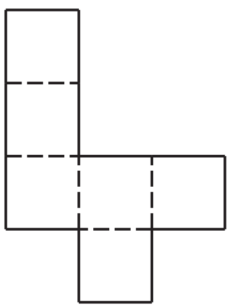
(i)

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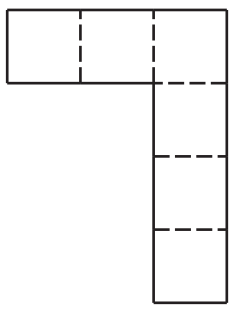
(j)

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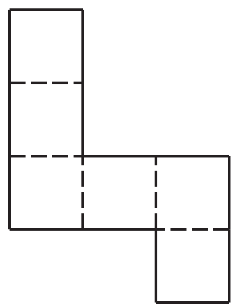
(k)

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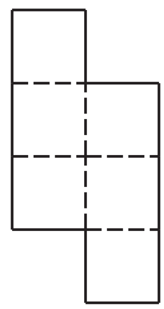
(l)

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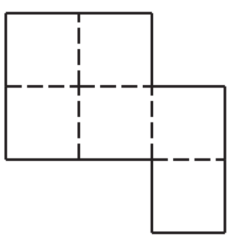
(m)

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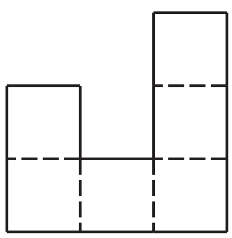
(n)

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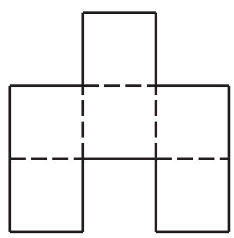
(o)

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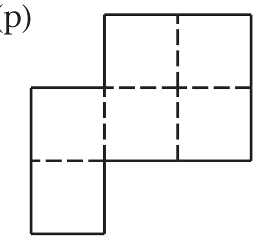
(p)

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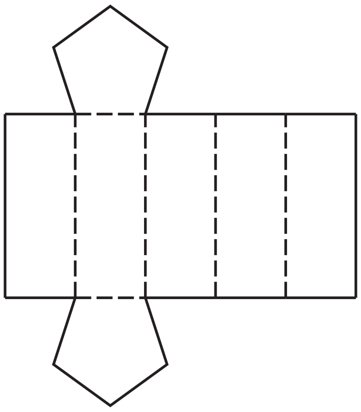
9. In each case below, state whether the diagram will work or not work as a net for making a pentagonal prism. The base need not be a regular pentagon. In the cases where the diagram will not work, explain why.

(a)



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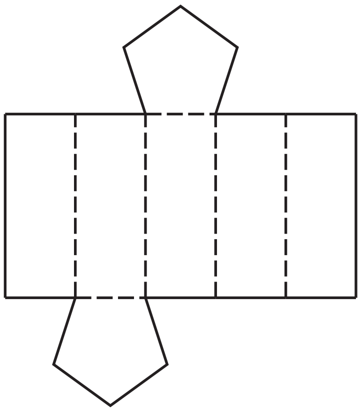
(b)



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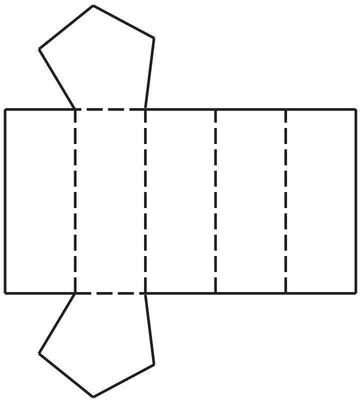
(c)



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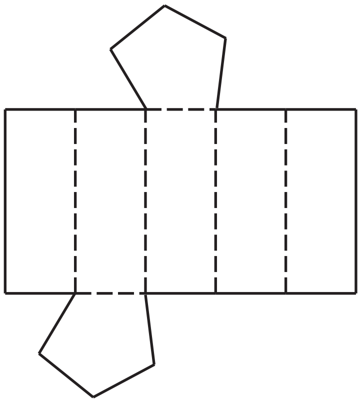
(d)



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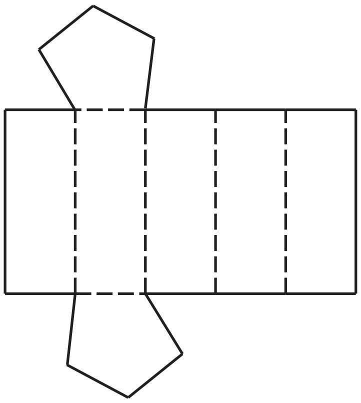
(e)



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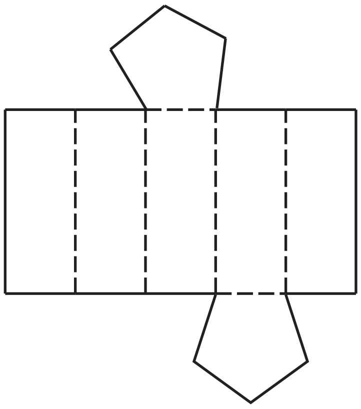
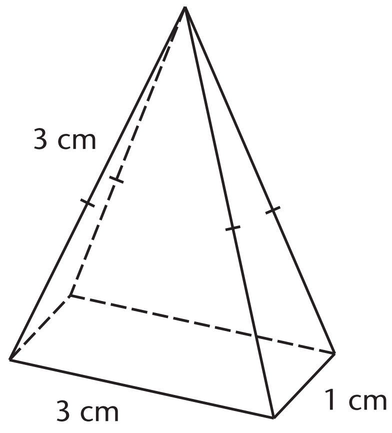
(f)

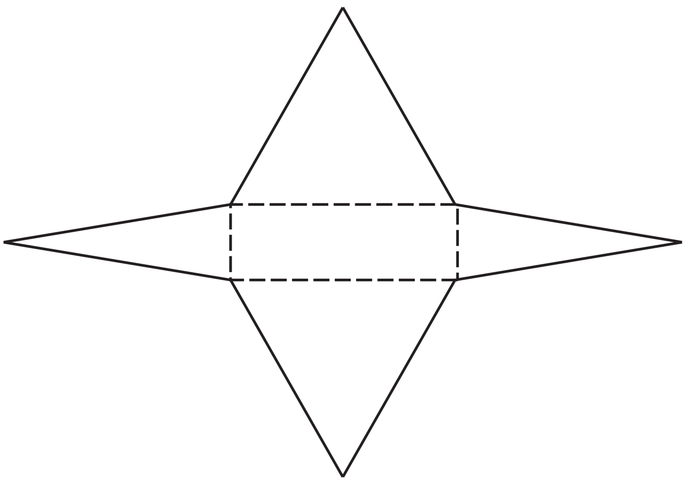


....

DRAWING NETS AND CONSTRUCTING 3D MODELS OF PYRAMIDS

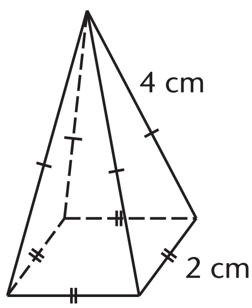
1. Write the measurements on the sides of the net given.



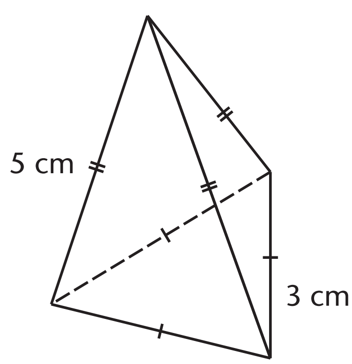


2. Draw accurate nets of the following pyramids.

   (a) Square-based pyramid



   (b) Triangular pyramid

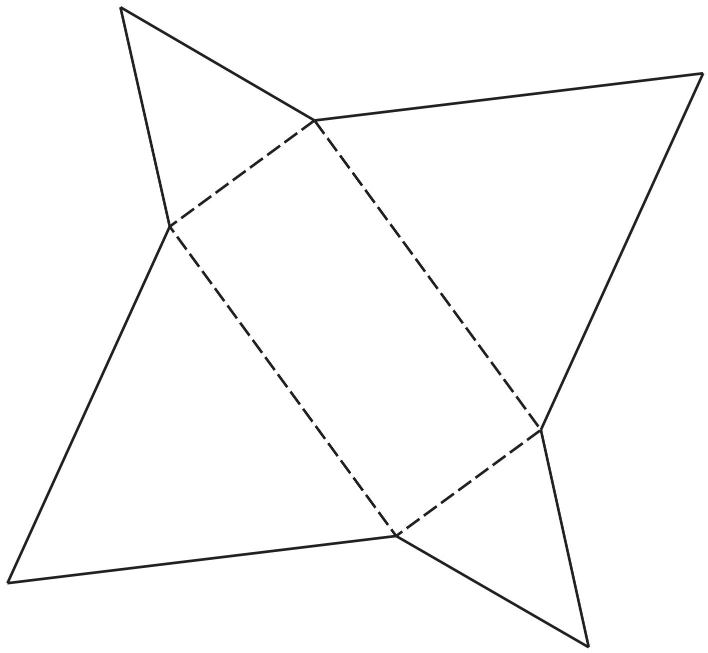


3. (a) Copy the nets you have drawn in question 2 onto cardboard or paper, but multiply the measurements by 2.
   (b) Then cut out, fold and paste the net to make a model of each 3D object.

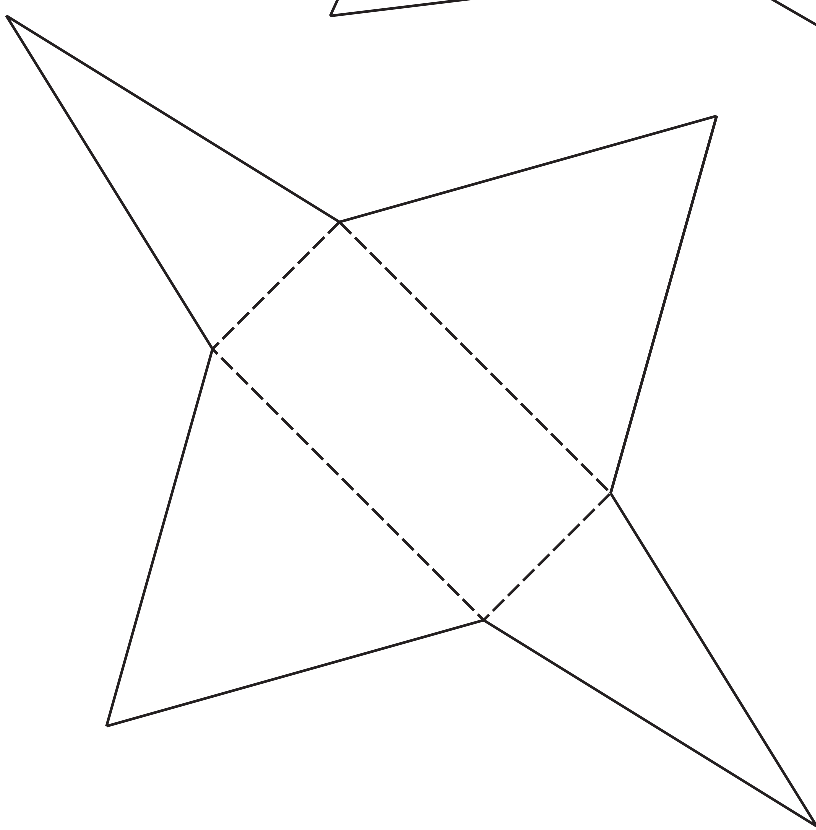
WHAT MAKES A NET WORK?

Which of the diagrams below will not work as nets to make a rectangular pyramid? You may have to take measurements to be sure in some cases, or make a copy and cut the diagram out and fold it. In each case where you say no, explain why you think it will not work.

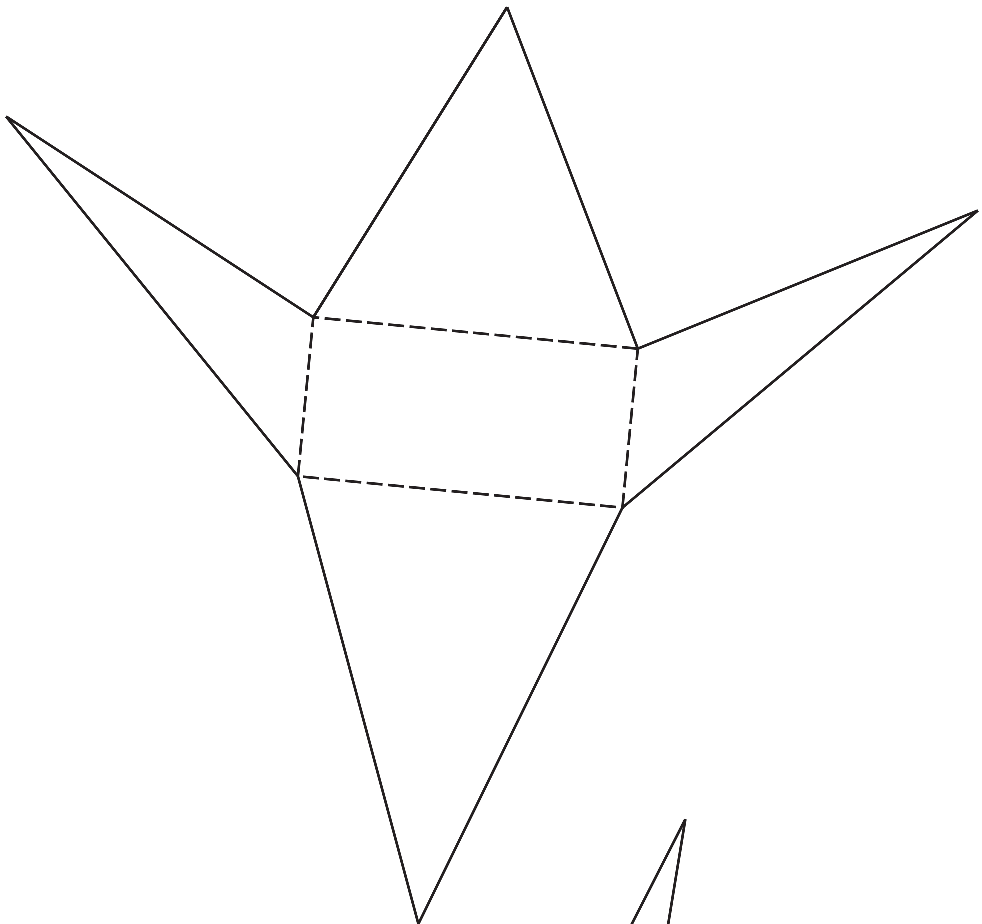
1.



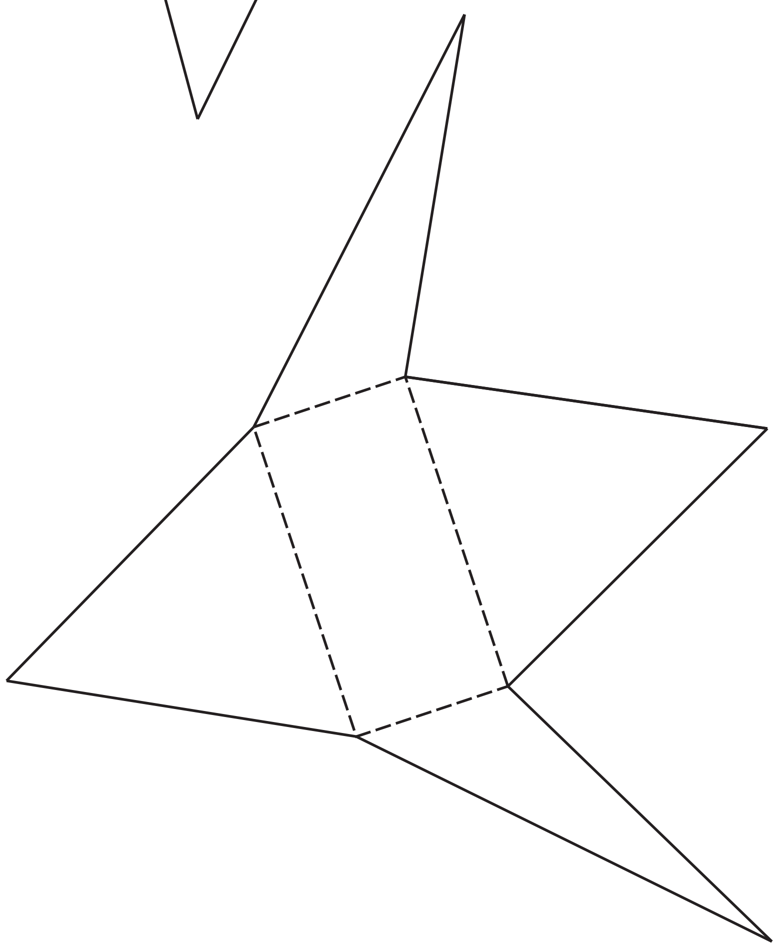
2.



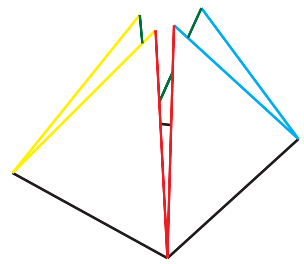
3.



4.



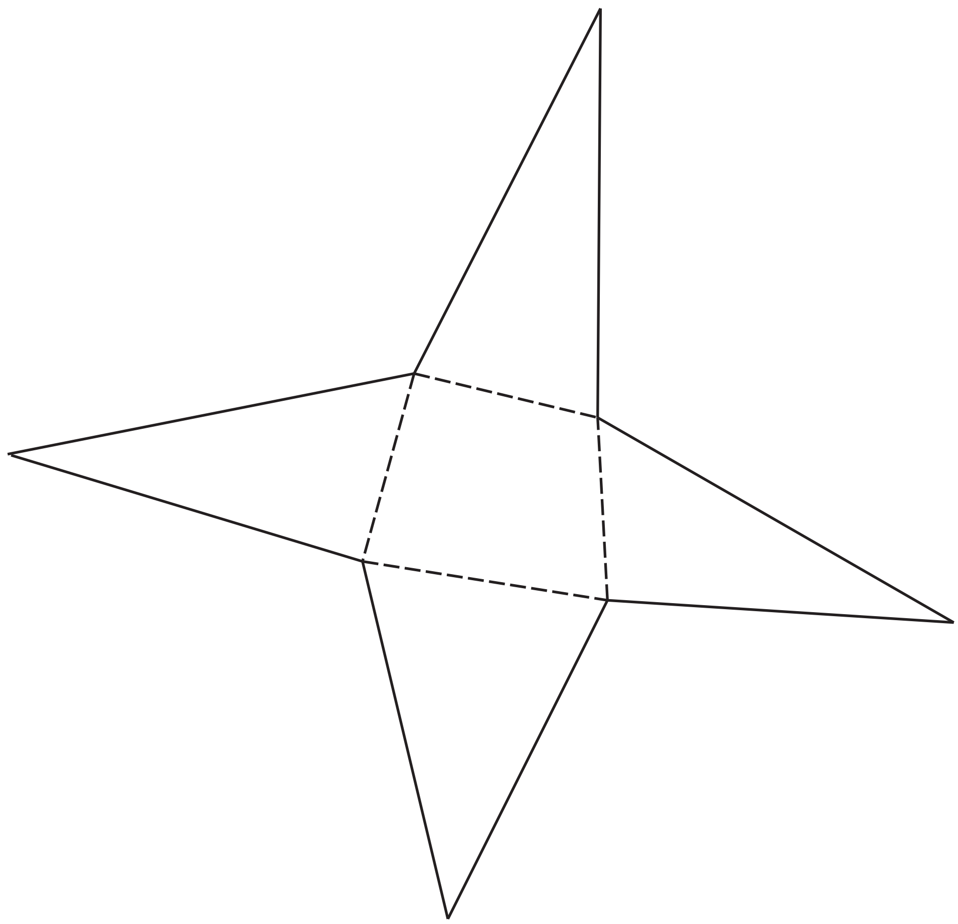
CIRCLES AND PYRAMIDS



In order to meet at the apex, one side of each triangular face of a pyramid must be the same length as the closest side of the triangle next to it.

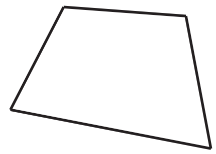
This means that certain line segments in the net of a pyramid must be equal.

1. Mark the line segments that should be equal in the diagram below, so that a pyramid can be made by folding a cut-out of the diagram on the dotted lines.



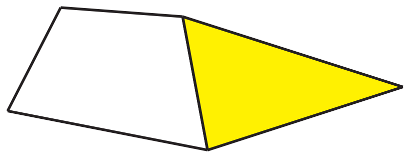
2. Make an accurate copy of the above diagram on stiff paper or cardboard. Cut it out and fold along the dotted lines. See if you can make a pyramid in this way.

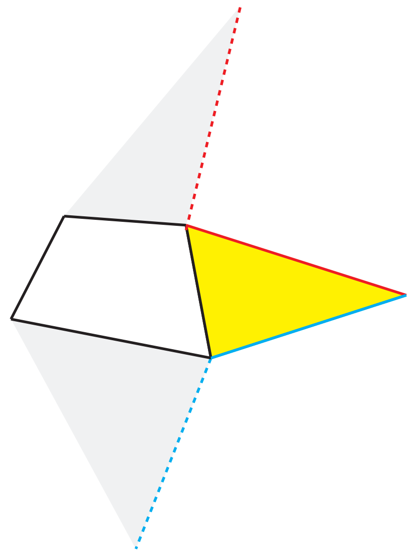
Any polygon can form the base of a pyramid.



If you want to draw a net for a pyramid, you can start by drawing a polygon.

Then draw any triangle on one side of the polygon.



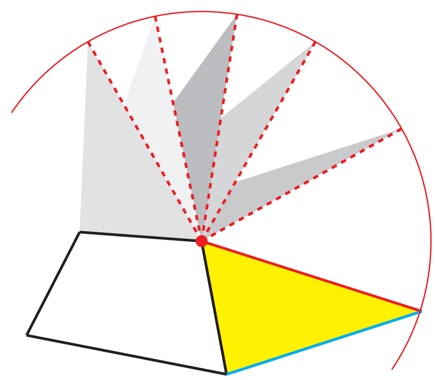


The triangles that will be adjacent to the first one must each have one side equal to the matching side of the first triangle, as indicated with the solid and dotted red and blue line segments in the sketch on the right.

The dotted line segments can be in other positions too, as long as they have the same lengths as the coloured sides of the first triangle.

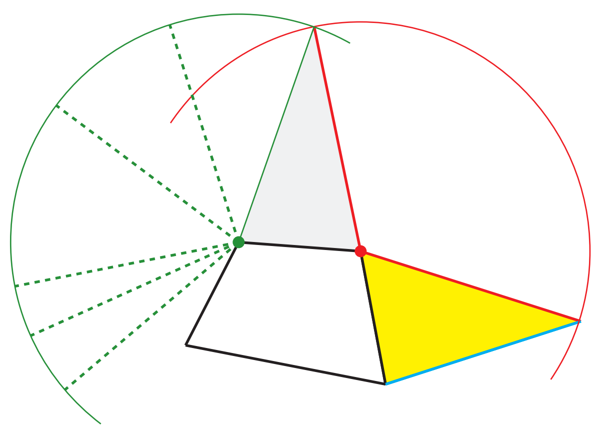
This means that once the first triangular face is drawn, there are many different possibilities for each of the two triangles that will be adjacent to it on the pyramid.

The circle with the red dot as midpoint, on the sketch on the right, shows the possibilities for one triangular face.



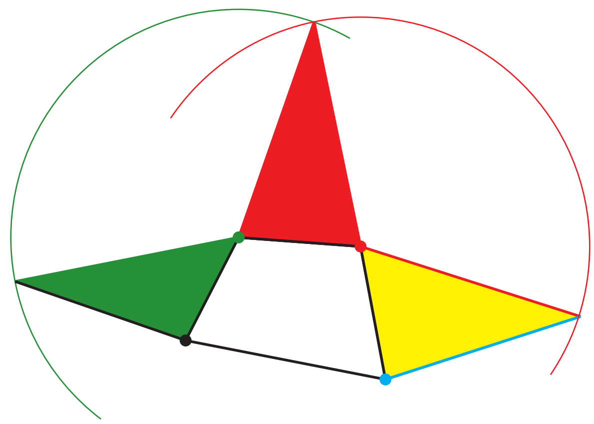
Once a triangle on the upper edge of the base is chosen, a circle can be drawn around the green vertex to indicate the possibilities for the third triangular face. The radius of this circle is the length of second leg of the second triangle, shown in solid green on the sketch.

Any line segment drawn from the green circle to the green vertex can be a side of the third triangular face.



Only one triangular face remains to be drawn now. We will refer to it as the blue triangle.

The black and blue dots on the sketch show where two vertices of the blue triangle should be.



3. Roughly draw the fourth triangular face for a pyramid on the above sketch. Think how long the sides should be so that the diagram will work precisely as a net to make a pyramid.

4. (a) How can the black dot and the green triangle help you to get some idea as to where the third vertex of the blue triangle should be?

....

....

....

....

(b) How can the blue dot and the yellow triangle help you to get some idea as to where the third vertex of the blue triangle should be?

....

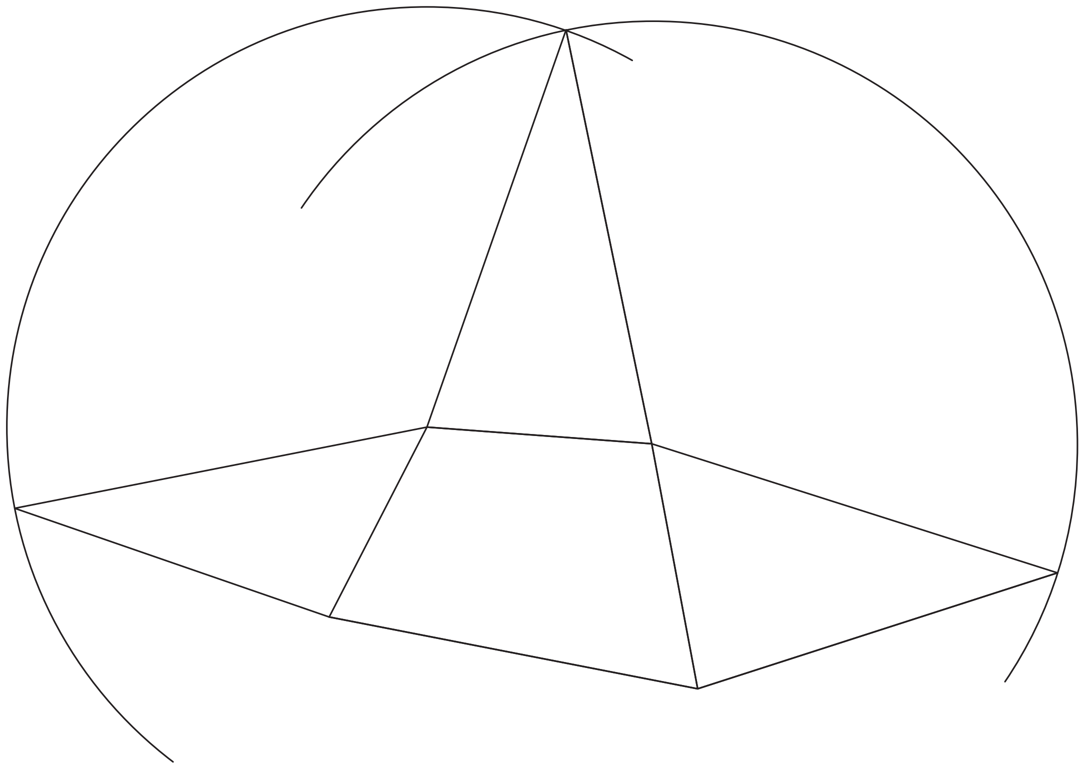
....

....

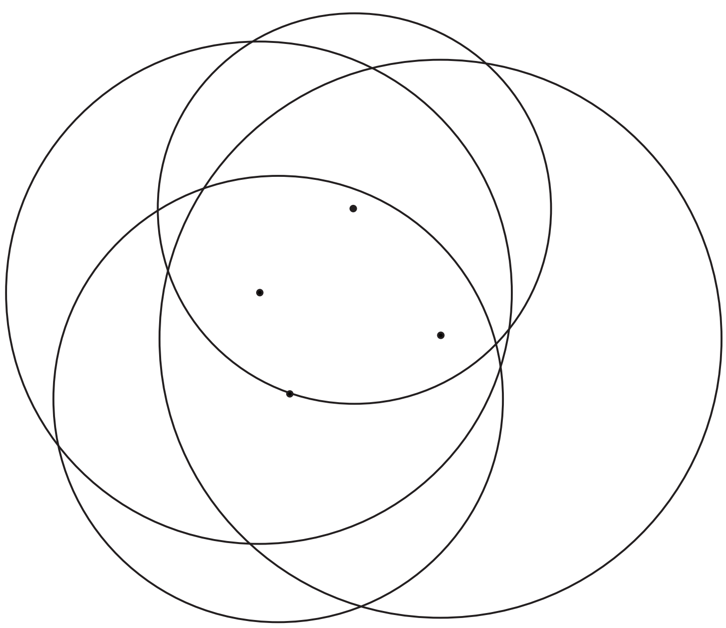
....

5. An enlargement of the sketch given at the top of this page is given on the next page. Use your pair of compasses to find the third vertex of the face that is not yet drawn, and complete the net for the pyramid. Then make a copy of the diagram on stiff paper or cardboard, cut it out and fold it to see whether it forms a pyramid.

Maths2_Gr8_LB_Book.indb 220 2014/09/04 10:39:53 AM



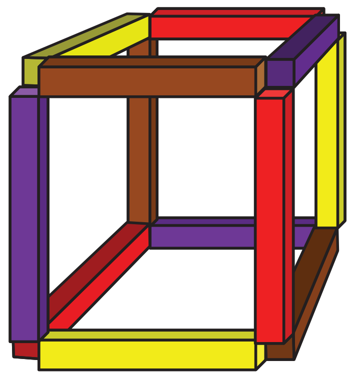
6. Join points on this sketch to draw a net for a pyramid.



13.3 Platonic solids

MAKING POLYHEDRA WITH IDENTICAL FACES AND EQUAL EDGES

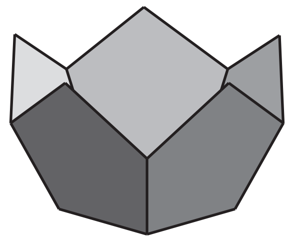
A cube is a special type of polyhedron. It has 6 identical faces, and its 12 edges are all equal in length.

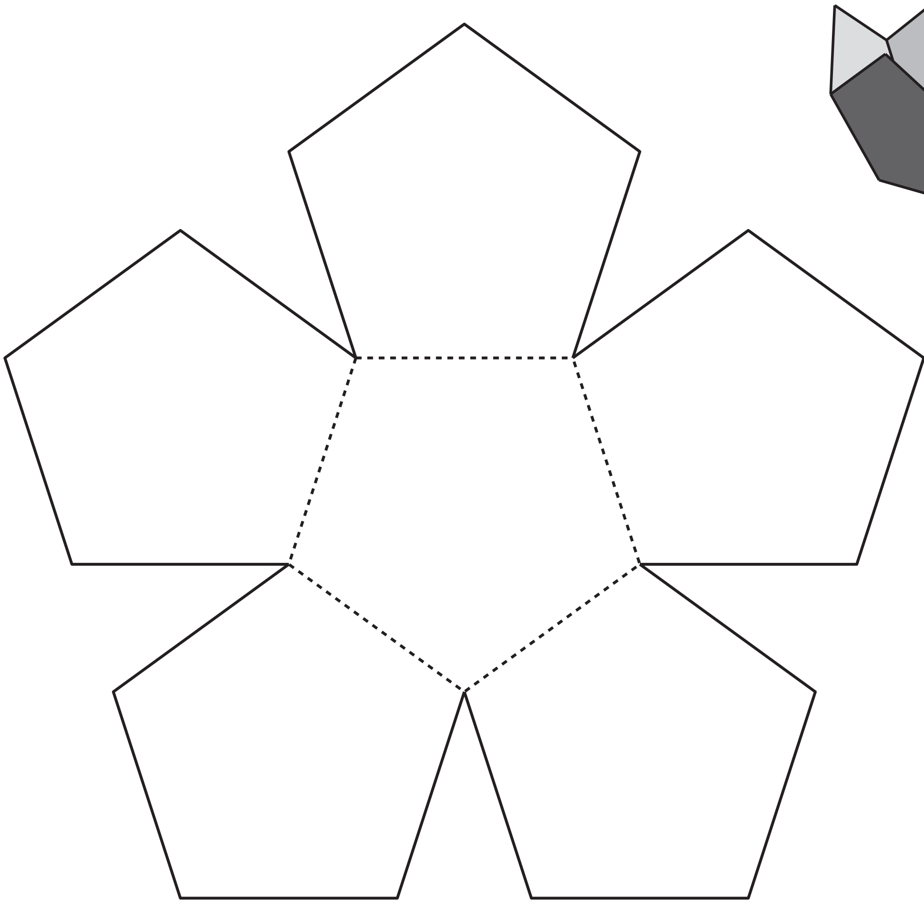


1. How many vertices does a cube have? ....
2. Can you think of an object which has faces that are identical triangles, and all its edges are equal? Try to draw a rough sketch of the net for such an object in your exercise book.

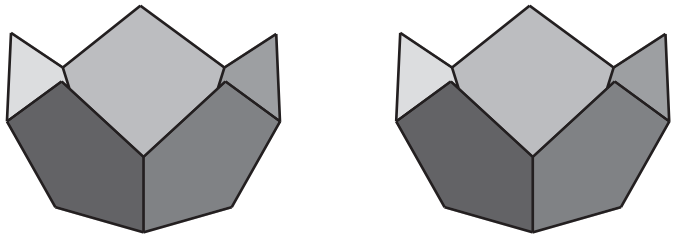
   ....

3. (a) Make a copy of the diagram below. Cut it out and fold it along the dotted lines. Attach the faces with sticky tape to make an open container with pentagonal faces.

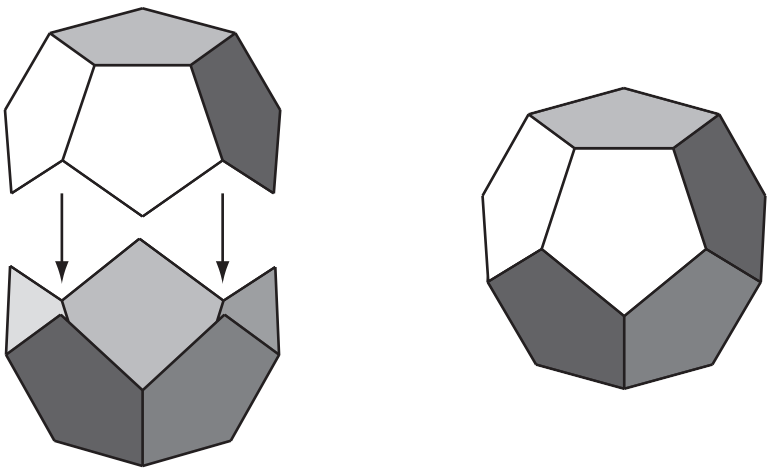




(b) Make another copy of the pentagon diagram, and make a second container.



(c) Turn one of your containers upside down and put the two together to form a polyhedron.



(d) How many faces does your polyhedron have? ....

(e) Are the faces identical, and what shape are they?

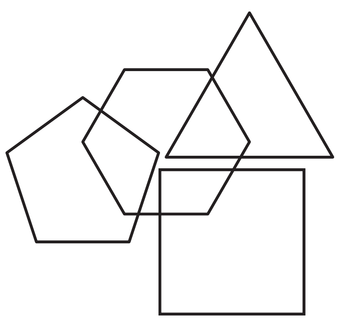
....

(f) Are the edges equally long, and how many edges are there?

....

A polyhedron of which all the faces are identical regular polygons is called a **Platonic solid**, because the Greek philosopher Plato was fascinated by such objects.

A **regular polygon** is a polygon with equal sides and equal angles.



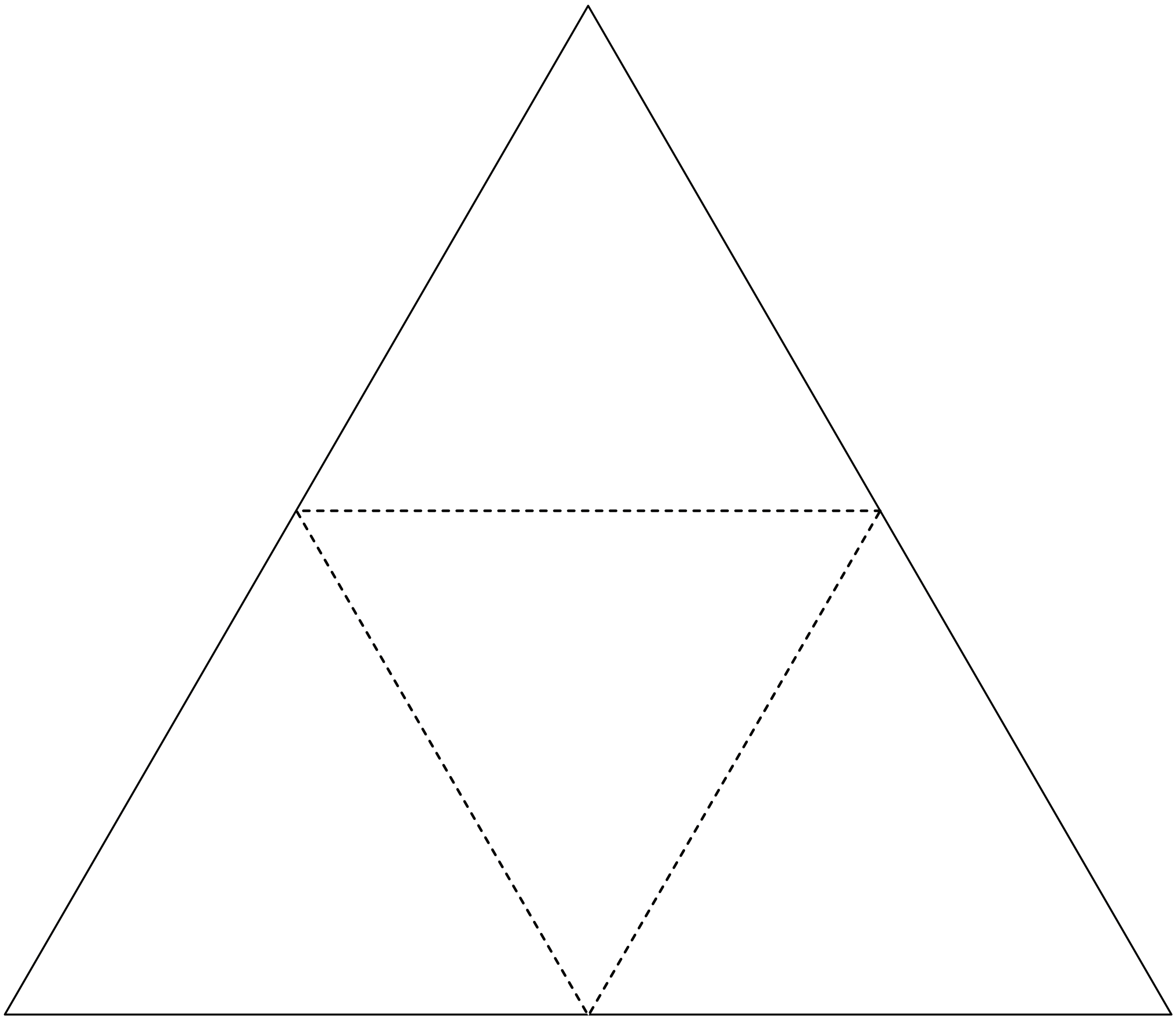
4. Do you think the diagram below can be used as a net to make a Platonic solid? If it is possible, how many faces, how many edges and how many vertices will the polyhedron have?

....

....

....

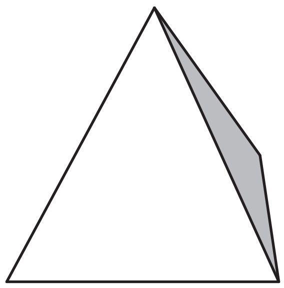
....



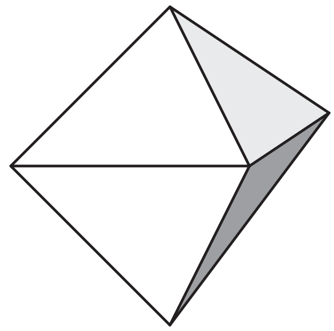
5. (a) Make two accurate copies of the above diagram. Cut out on the solid lines and fold on the dotted lines to form two identical polyhedra. Polyhedra like this are called **regular tetrahedra**.
   (b) Try to combine your two regular tetrahedra to make another solid, with 6 faces, 9 edges and 5 vertices.

THE PLATONIC SOLIDS

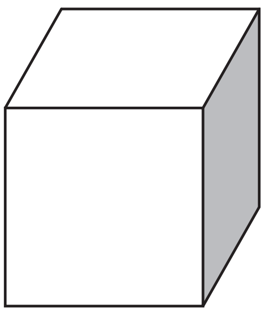
The Platonic solids have special names, and these are given below. There are only five Platonic solids.



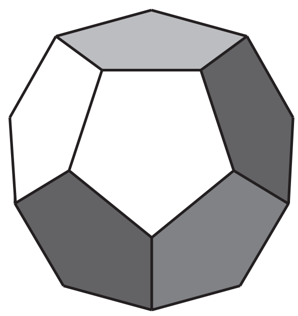
A **tetrahedron** consists of 4 equilateral triangles. It has 6 edges and 4 vertices.



An **octahedron** consists of 8 equilateral triangles. It has 12 edges and 6 vertices.

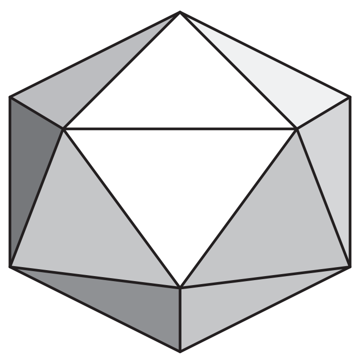


A **hexahedron** (also known as a **cube**) consists of 6 squares. It has 12 edges and 8 vertices.



A **dodecahedron** consists of 12 regular pentagons. It has 30 edges and 20 vertices.

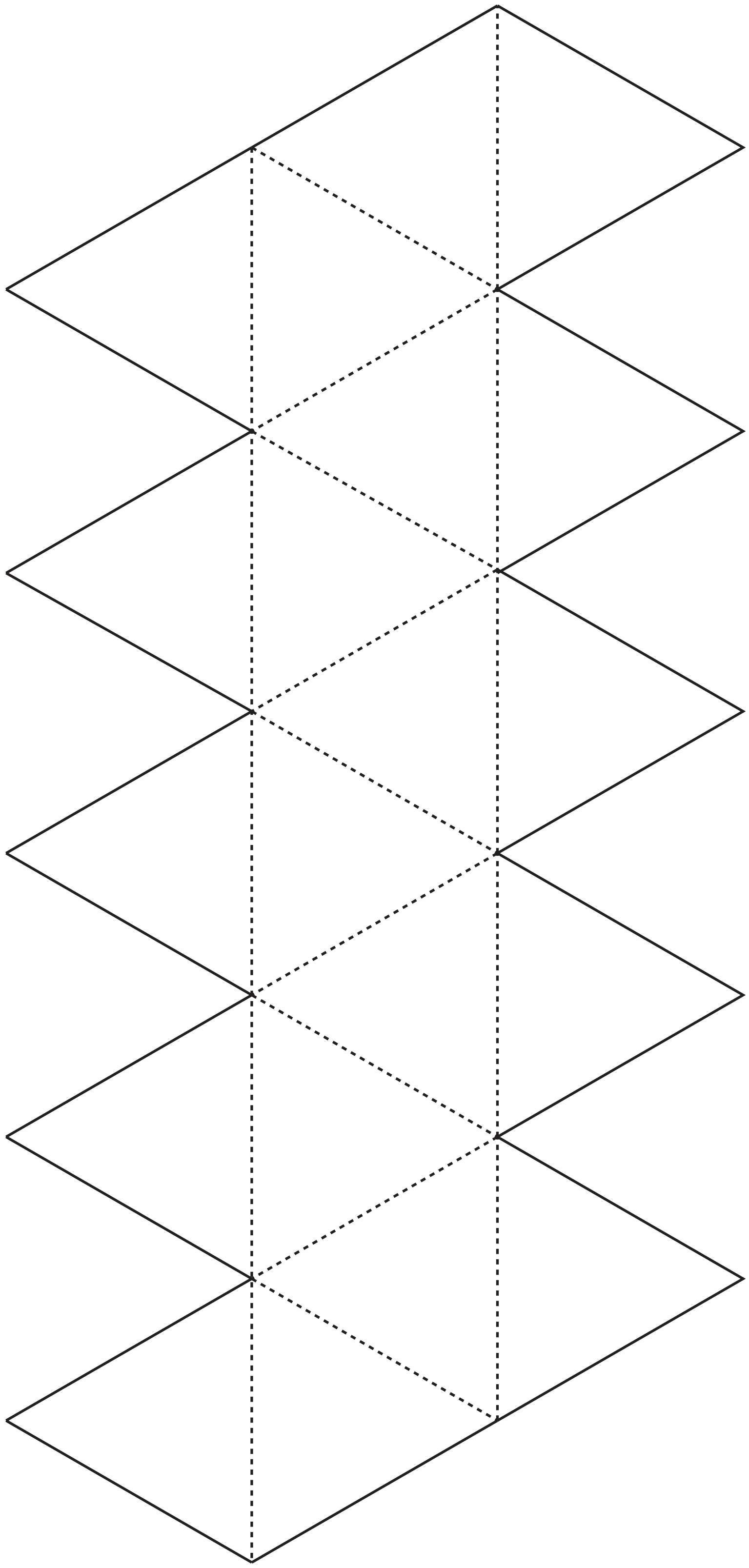
An **icosahedron** consists of 20 equilateral triangles. It has 30 edges and 12 vertices.



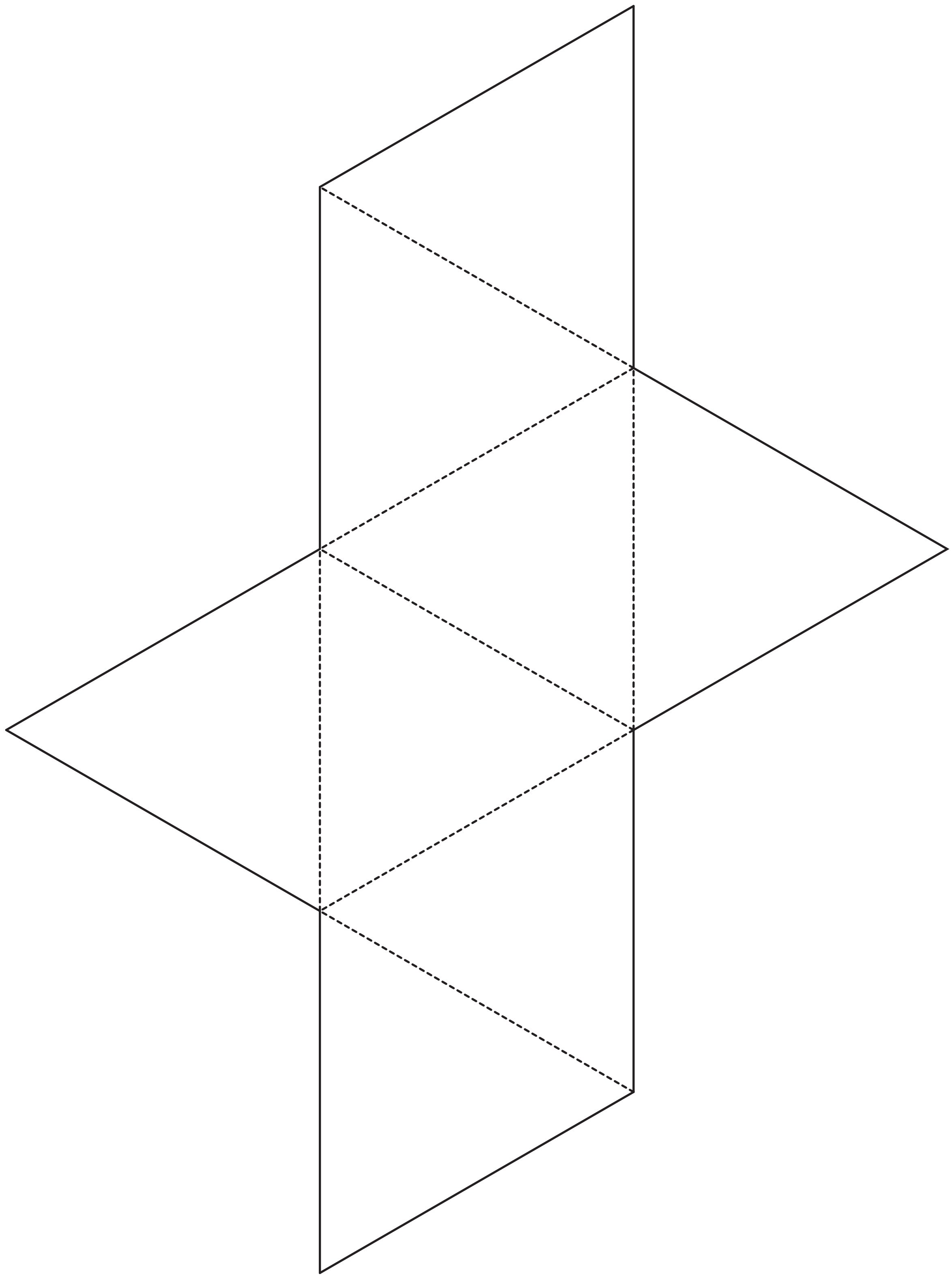
1. Nets for some of the Platonic solids are given on the following pages. Write the names of the objects next to the nets that can be used to make them.

2. Investigate whether Euler's formula is true for the Platonic solids. ....

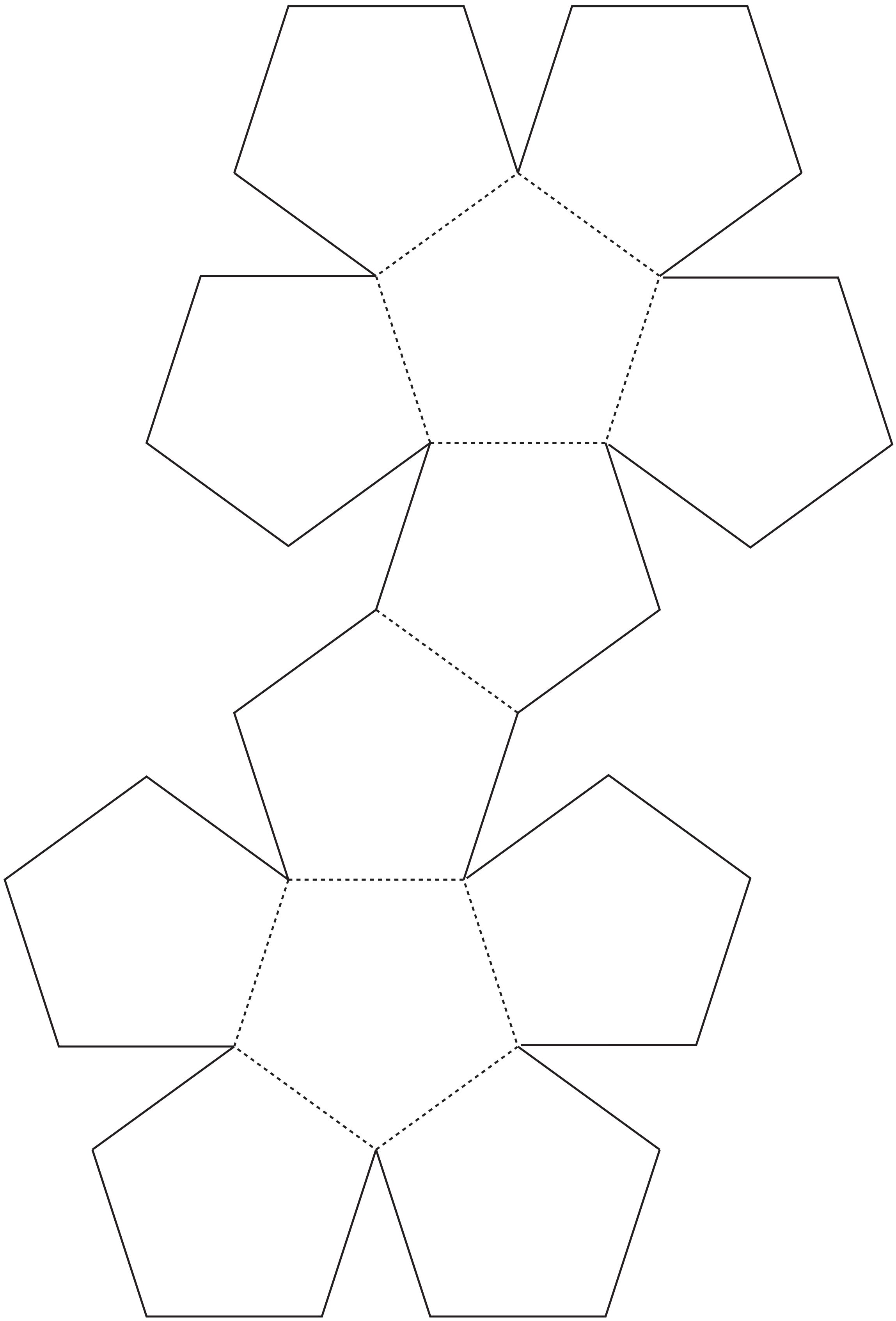
(a) ....



(b) ....



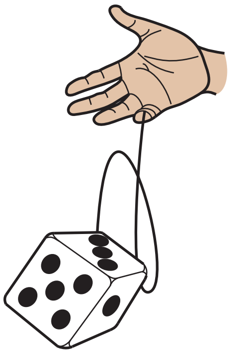
(c) ....



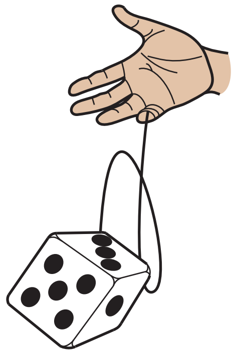
Maths2_Gr8_LB_Book.indb 228 2014/09/04 10:39:55 AM

CHAPTER 14
Probability

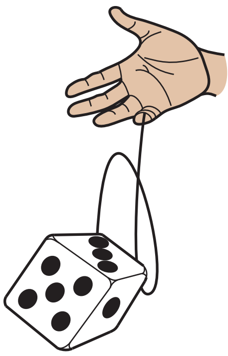
Some actions, like drawing a card from a bag with ten different coloured cards, have different possible results (outcomes). One cannot know what card will be drawn. You will learn that some predictions can be made though about what will happen if the action is repeated many, many times.



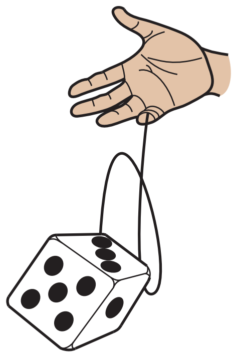
5?



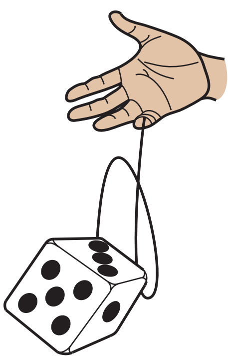
3?



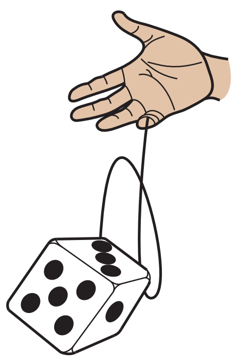
2?



6?



4?



1?

14 Probability

14.1 How often different things can happen

DIFFERENT FRACTIONS OF A WHOLE NUMBER

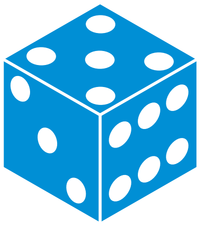
Jayden lives close to the sea. He goes fishing every day. Some days he catches no fish, but on some days he catches several fish. He never catches more than five fish in a day.

He has decided that he will always stop fishing when he has caught five fish in one day.

1. What are the different possible outcomes of each of Jayden's daily fishing trips?

....

2. Jayden rolls a dice just once each day before he goes fishing.



   (a) What are the possible outcomes of rolling a dice once?

   ....

   (b) Are the six outcomes of rolling a dice equally likely?

   ....

   (c) Is there any reason for Jayden to believe that the outcome of his fishing trip on a day will be one less than the number that came up when he rolled the dice on that day?

   ....

3. Jayden keeps a record of the outcomes of his daily dice rolls. Here is a summary of his record for 60 consecutive days.

Outcome	1	2	3	4	5	6
Frequency	9	9	12	11	10	9

   (a) How many times was the outcome a 6? ....

   (b) What fraction is this of the total of 60 events? ....

   (c) On what fraction of the days was the outcome a 3? ....

The fraction of a number of events which have a specific outcome is called the **relative frequency** of that outcome.

Maths2_Gr8_LB_Book.indb 231 2014/09/04 10:39:55 AM

4. What is the relative frequency of a:
   (a) 5 in Jayden's series of 60 dice rolls? ....
   (b) 4 in Jayden's series of 60 dice rolls? ....

> The **range** of a set of numbers is the difference between the smallest and largest numbers in the set.

5. What is the **range** of the relative frequencies of the different outcomes in Jayden's series of dice rolls? Express the range as a fraction in sixtieths, and as a percentage.

   ....

6. Do you think the six possible outcomes of Jayden's daily fishing trips are equally likely? Give reasons for your answer.

   ....

   ....

   ....

   ....

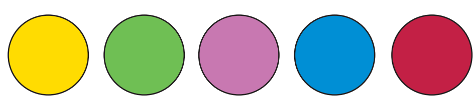
7. (a) Jayden has a book in which he keeps a record of the outcomes of his daily fishing trips. A summary of his record for a period of 200 consecutive days is given in the table below. Write the relative frequencies of the different outcomes in the table, with each expressed as two hundredths.

Outcome	0	1	2	3	4	5
Frequency	30	32	68	54	12	4
Relative frequency						

   (b) What is the range of the relative frequencies in this case? Express the range as a common fraction and as a percentage.

   ....

HOW OFTEN CAN WE EXPECT SOMETHING TO HAPPEN?



Imagine that you have five coloured buttons as shown above in a paper bag.

1. Imagine that you put your hand into the bag without looking inside, and grab one of the buttons.
   (a) Can you say which colour that button will be? ....
   (b) Discuss this with some classmates.

2. (a) What are the different possible colours of buttons that you could draw from the bag?

....

(b) How many different possibilities are there? ....

3. Read the passage below, then answer the questions that follow.

*When you draw a button from the bag, we say you perform a **trial**. The colour you draw is called the **outcome** of the trial.*

(a) What are the different possible outcomes of the trial if you draw one button out of the bag?

....

(b) Imagine that you put the first button back into the bag. If you now draw one button from the bag again, what are the possible outcomes of this new trial?

....

(c) Imagine that you repeat the event a third time. What are the possible outcomes of this new trial?

....

(d) Imagine that you perform many trials. What are the possible outcomes of each repetition?

....

4. (a) When you draw one of the five buttons many times and put it back each time, do you think you will draw one colour more often than the others?

....

(b) Discuss this with some classmates.

5. (a) Imagine that you draw a button out of the bag with five buttons and put it back, and repeat this 60 times. Approximately how many times do you think you will draw the red button?

....

(b) Approximately how many times do you think you will draw the pink button?

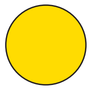
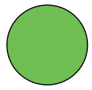
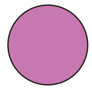
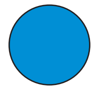
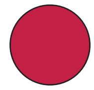
....

(c) Discuss this with some classmates.

When there is no reason to believe that any outcome will occur more often than any other outcome, the outcomes are said to be **equally likely**.

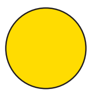
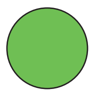
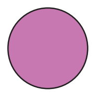
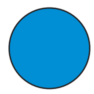
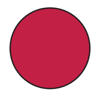
6. Susan decides to perform 160 trials on the bag with five buttons. In each trial she will draw one button from the bag, note its colour, and put it back. Lebogang decides she will perform 60 trials and Archie decides to perform 40 trials.

   Approximately how many times do you think each of them will draw each of the buttons? Enter your expectations in the table below.

					
Susan					
Lebogang					
Archie					

7. Here are the answers that eight different people gave for Archie, with his 40 trials.

> "Close to 6" means it can be 6 or another number close to 6, for example 5 or 7 or 4 or 8.

					
Answer A	close to 5	close to 5	close to 5	close to 5	close to 20
Answer B	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9
Answer C	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8
Answer D	close to 7	close to 9	close to 8	close to 10	close to 6
Answer E	close to 6	close to 6	close to 6	close to 6	close to 6
Answer F	6	9	7	8	10
Answer G	8	8	8	8	8
Answer H	close to 8	close to 8	close to 8	close to 8	close to 8

Which answers do you think are good answers, and which do you think are poor answers? For each answer explain why you think it is good or poor.

8. (a) How much is 1 fifth of 160, 1 fifth of 60, and 1 fifth of 40? ....

   (b) Look at your own answers for question 6 again. Do you still agree with your answers? If you want to give different answers now, do so and explain what made you change your position.

   ....

9. Willem has decided to perform as many trials as he can in an afternoon, drawing one button each time out of the bag with five coloured buttons.

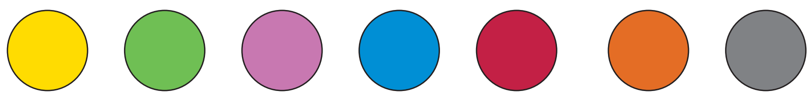
   (a) In close to what fraction of the trials can he expect to get yellow as the outcome?

   ....

   (b) In close to what fraction of the trials can he expect to get red as the outcome?

   ....

10. Manare has decided to perform as many trials as he can in an afternoon, drawing one button each time out of the bag with seven different coloured buttons.



   (a) In close to what fraction of the trials can he expect to get blue as the outcome?

   ....

   (b) In close to what fraction of the trials can he expect to get grey as the outcome?

   ....

11. Miriam has decided to perform as many trials as she can in an afternoon, drawing one button each time out of a bag with twelve different coloured buttons.
    In close to what fraction of the trials can Miriam expect to get each specific colour as the outcome? ....

The number of times that a specific outcome is obtained during a series of trials is called the **frequency** of the outcome.

12. What is the frequency for each of the following colours in answer F, in question 7 on the previous page?

   (a) red .... (b) pink ....

   (c) yellow .... (d) blue ....

When the different possible outcomes of an event are equally likely, it is reasonable to expect that when the event is repeated many times, the frequencies for the different outcomes will be almost equal.

AN INVESTIGATION

1. Make eight small cards or pieces of paper. On each card write a different letter. Use the letters A, B, C, D, E, F, G and H. Put the cards in a paper bag. Imagine that you draw a card out of the bag, note the letter and put it back. Imagine that you perform 40 such trials, noting the outcome each time. Then you find the frequency for each letter. To what number do you think each of the frequencies will be close? ....
   The number you think of may be called the **expected frequency**.

2. What will be the expected frequencies for each letter if:
   (a) 200 trials are performed? ....
   (b) 1 000 trials are performed? ....

3. Now actually do the experiment described in question 1. Record your results with tally marks in the table below. When you have finished, count the tally marks to find the **actual frequencies**.

	A	B	C	D	E	F	G	H
Tally marks								
Actual frequency								
Expected frequency	5	5	5	5	5	5	5	5

4. Write your actual frequencies on a slip of paper, in a table like this.

	A	B	C	D	E	F	G	H
Actual frequency								

5. The next step is to collect the slips of four different classmates, and write their frequencies in rows 1, 4, 7, 10 and 13 of the table on the next page, together with your own frequencies. **Do not do it yet**. When you put the five sets of results together, and add them up, you will have the actual frequencies out of 200 trials. You will write these in row 16. In the row for expected frequencies, write the numbers to which you think the frequencies will be close.

6. Now work with your four classmates, and complete rows 1, 4, 7, 10 and 13.

7. In the first empty row after each actual frequency row, express the frequency as a fraction of the total number of outcomes in the experiment, which was 40 in each case. You need not simplify the fractions in rows 2, 5, 8, 11 and 14 of the table.

8. In rows 17 and 20, express the frequencies of rows 16 and 19 as fractions of 200.

9. In the remaining empty rows, express the fractions as percentages.

10. Calculate the ranges of the numbers in rows 3, 6, 9, 12, 15 and 18.

Row 3: ....

Row 6: ....

Row 9: ....

Row 12: ....

Row 15: ....

Row 18: ....

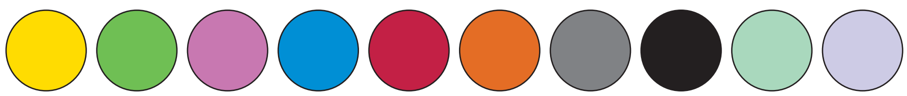
		A	B	C	D	E	F	G	H
1	Actual frequencies								
2									
3									
4	Actual frequencies								
5									
6									
7	Actual frequencies								
8									
9									
10	Actual frequencies								
11									
12									
13	Actual frequencies								
14									
15									
16	Total actual frequencies								
17									
18									
19	Expected frequencies								
20									
21									

11. In which row is the range the smallest? Try to explain why this is the case.

12. In which of the rows in question 10 are the numbers closest to the expected percentages in row 21?

The expected relative frequency of an outcome is called the **probability** of the outcome.

13. Imagine that you have ten different coloured buttons in a bag, as shown below.



(a) Imagine that you draw one button out of the bag.
How many different equally likely outcomes are there for this trial?

(b) Imagine you draw one button out of the bag, look at the colour and make a tally mark in the column for that colour on the table below. Imagine you do it many times. Approximately what fraction of the total number of tally marks do you expect to be in each column?

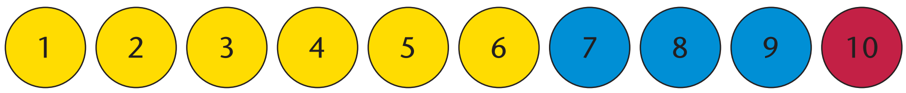
(c) The fraction you have specified in (b) is the probability of the outcome for each of the columns. Would you expect to get precisely that fraction in each column?

(d) Hashim says he expects to have **approximately** 10 tally marks in each column, because the outcomes are equally likely. Do you agree with Hashim? Give reasons for your answer.

14.2 Probability

In this activity you have to think about the following situation.

There are 10 coloured, numbered buttons in a bag: 6 yellow buttons, 3 blue buttons, and 1 red button.



1. (a) What fraction of the total number of buttons is yellow?

   (b) What fraction of the total number of buttons is blue?

   (c) What fraction of the total number of buttons is red?

2. Suppose you put your hand into the bag without looking inside, take one button out and note its colour, and then put it back into the bag.

   If you repeat this trial many times, you will sometimes get a yellow, sometimes a blue and sometimes a red button.

   (a) Do you think you will get blue more often than yellow? Explain your answer.

   (b) Do you think you will draw yellow about twice as often as blue?

   (c) Can you be certain which colour will be drawn? Explain your answer.

   (d) Share your ideas with two classmates.

Here is an experiment that you will do later. **Do not do it now**.

Put 10 buttons like those on page 239, or pieces of paper or cardboard with the names of the colours written on them, in a bag. Put your hand into the bag without looking inside, and take one button out. Check what colour it is, make a tally mark in the column below for that colour, and put the button back into the bag. Do this 10 times.

Yellow	Blue	Red

Each time you perform a trial, a certain **event** takes place, and there are three possible events:

A. The event of the colour being yellow
B. The event of the colour being blue
C. The event of the colour being red

3. (a) In how many different ways can event A be achieved in one trial? ....
   (b) In how many different ways can event B be achieved in one trial? ....
   (c) In how many different ways can event C be achieved in one trial? ....

4. (a) Suppose you do the experiment, and make 10 trials. Do you think event A will happen 3 times or maybe 4 times, event B will happen 3 times or maybe 4 times and event C will happen 3 times or maybe 4 times?

   ....

   ....

   (b) Share your ideas with two classmates.
   (c) Do you rather think event A will happen 6 times (or maybe 5 or 7 times), event B will happen 3 times (or maybe 2 or 4 times), and event C will happen once (or maybe twice or not at all)?

   ....

   ....

   ....

   ....

   (d) Share your ideas with two classmates.

5. (a) Do the experiment that is described before question 3, and write the results in the second row of the table on the next page.
   (b) Repeat the experiment, and write the results in the third row of the table.
   (c) Repeat the experiment three more times, and enter the results in the table.
   (d) Complete the last two rows of the table.

Outcome	Yellow	Blue	Red
Frequency of each colour during the first 10 trials			
Frequency of each colour during the second 10 trials			
Frequency of each colour during the third 10 trials			
Frequency of each colour during the fourth 10 trials			
Frequency of each colour during the fifth 10 trials			
Total frequencies out of 50 trials			
Total frequencies divided by 5			

When you did the experiment for the first time in question 5(a), you performed 10 **trials**: you took a button out of the bag, and inspected the colour.

Each time, there were three **possible outcomes** for the trial: the button could be **yellow**, it could be **blue** or it could be **red**.

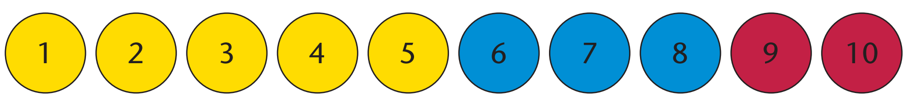
We can also say that three different events were possible: yellow, blue and red. But if we consider the numbers on the buttons, 10 different outcomes are possible.

6. (a) How many different outcomes (numbered buttons) will produce the event yellow? ....
   (b) How many different outcomes will produce the event blue? ....
   (c) How many different outcomes will produce the event red? ....

7. (a) What fraction of the ten possible outcomes will produce the event yellow? ......
   (b) What fraction of the ten possible outcomes will produce the event blue? ....
   (c) What fraction of the ten possible outcomes will produce the event red? ....

The fractions you have given as answers for question 7 are the **probabilities** of the three different events.

8. (a) What is the probability of getting blue when one of the buttons below is drawn from a bag? ....



   (b) Describe in your own words what is meant by saying the probability of an event is $\frac{3}{20}$.

....

....